

PDE in Finance, Spring 2009
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Homework 2

1. Corridor option A *corridor option* is a contingent claim that pays the holder a pre-determined amount of money if the underlying asset remains in a specified range for a given period of time. The payoff is made at the maturity date. If the underlying asset exits the range, the payoff is zero and the contract expires.

Assume that 1 EUR = 1.2750 USD, and that the 3-month interest rates are $r_{US} = 0.48\%$, $r_{EUR} = 0.95\%$. Using a trinomial tree (finite-difference scheme) find the fair value of an 90-day corridor option that has a payoff of USD 10 Million if the exchange rate stays in the range $1.24 < USD/EUR < 1.30$. Assume that the volatility is $\sigma = 20.3\%$.

Derive an alternative pricing scheme by constructing the fundamental solution of the boundary-value problem using the technique of Jacobi Theta-functions.

Give a table of fair values for the option corresponding to different values of the volatility, going from 15% to 25%. Notice that the value is a decreasing function of volatility. Why is this so?

2. Down and out Call Using the data of the previous exercise, price a 90-day down-and-out EUR Call/ USD Put with strike $K = 1.30$ and knock-out barrier $L = 1.23$, (i) using a trinomial tree and (ii) using the fundamental solution constructed by the method of images.

3. Convexity of down and out calls Consider a generic down and out call on an asset, with strike K and lower KO barrier L . Let $V(S, t)$ denote the fair value of the option. Is $S \rightarrow V(S, t)$ a convex function of S ? Analyze the problem for different values of the dividend yield, interest rate and volatility, the strike price and the barrier.

Give a financial interpretation of the option for the case $K \leq L$.

4. Vega Derive a PDE for the Vega $= \frac{\partial V}{\partial \sigma}$ of a European-style contingent claim, as a function of strike and time to expiration (and all other parameters). Deduce that derivatives with convex payoffs have positive vol exposure and derivatives with concave payoffs give rise to negative volatility exposure. Write down the explicit form of Vega for a European option. For which values of the strike is Vega the largest/smallest. Relate this to the PDE.

5. Range Accrual A derivative security pays the holder EUR 50,000 every day as long as EUR/USD is in the range \$1.22 - \$1.30. The maturity is 90 days. Once the exchange rate breaks the band, the payments will stop henceforth. Using the parameters from Problem 1, find the fair value of this option. Analyze its sensitivity to volatility and changes in interest rates.