Mathematics of Finance, Fall Semester 2017
Final Examination

Note: This exam requires the use of a calculator/program for mean-variance optimization.

1. Consider the following stocks A, B, C with the following characteristics

Stock	Exp. Return	Volatility
A	10%	20%
B	9%	10%
C	22%	40%

Correlations			
	A	B	C
A	1	0.43	0.25
B	0.43	1	0.04
C	0.25	0.04	1

Assume that you want to target a return of 15% per year by investing in the three stocks. What would be the portfolio which achieves such return and has the smallest variance? (Set up the optimization problem that you want to solve first, with letters; then solve numerically. Display the weights and the variance of the optimal portfolio).

2. Calculate the Implied Volatility and the Delta of a call option on Index XYZ, with the following characteristics:

   Underlying security: XYZ= $266.26
   Strike = $270.00
   Expiration: 197 days (1 day = 1/365 years).
   Interest rate: 1.75%
   Dividend yield: 1.90%
   Style: European option
   Price: $6.60

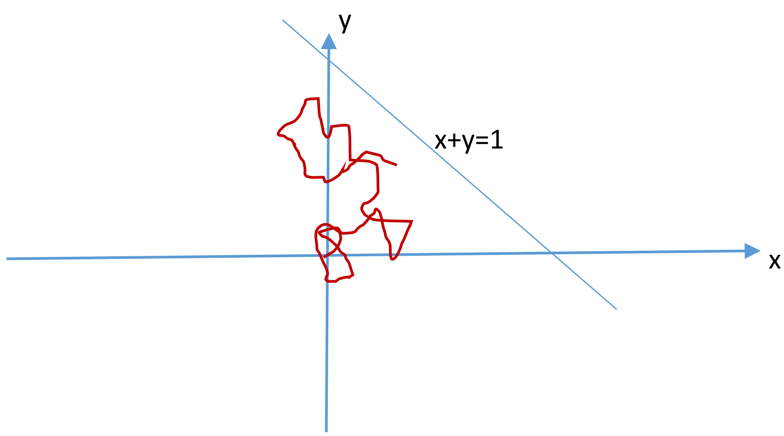
3. The returns of two stocks are modelled using linear combinations of Gaussian random variables. Let N_1, N_2, N_3, N_4 be independent, identically distributed standard normal (N(0,1) iid). Denote the returns of the stocks as X, Y and assume that:

$$X = \quad (0.2)N_1 + (0.2)N_2 + (0.1)N_3 + 0.12$$

$$Y = -(0.1)N_1 + (0.3)N_2 + (0.2)N_4 + 0.25$$

Compute the expected returns and the covariance matrix of X and Y. What is the expected return of the portfolio containing the two stocks which has minimum variance?

4. Let $W_1(t), W_2(t)$ be two independent standard Brownian motions, starting at zero. Viewing $\{(W_1(t), W_2(t)), t > 0\}$ as a random walk on the x-y plane starting at the origin, calculate the probability that the random walk never crosses the line x+y=1 for $0 \leq t \leq 3$. (see figure).



[Hint: show that $Z(t) = W_1(t) + W_2(t)$ is a *1-dimensional* BM (with variance not necessarily equal to 1) and use this.]