COMMON FACTORS AFFECTING BOND RETURNS

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Market participants have long recognized the importance of identifying the common factors that affect the returns on U.S. government bonds and related securities. To explain the variation in these returns, it is critical to distinguish the systematic risks that have a general impact on the returns of most securities from the specific risks that influence securities individually and hence have a negligible effect on a diversified portfolio.

We may use duration analysis to estimate how a change in the general level of interest rates affects prices of fixed-income securities. Practitioners are aware, however, that in many episodes yields have changed in ways not fully described by saying only that "yields went up" or "yields went down."

In this article we use an alternative approach, employing empirical research to determine the common factors that have affected returns on Treasury-based securities in the past. Our analysis suggests that most of the variation in returns on all fixed-income securities can be explained in terms of three "factors," or attributes of the yield curve, which we will call *level*, *steepness*, and *curvature*.

The three-factor approach presented here is especially useful for hedging. By considering the effect on a portfolio of each of the three factors, investors can achieve a better hedged position than they can get simply by holding a zero-duration portfolio. Because the three factors explain almost all the return variability across the whole maturity spectrum, this approach allows investors to hedge securities with instruments that may not fall in the same sector.

The portfolio in Table 1, consisting of three Treasury bond positions established on February 5, 1986, and sold on March 5, 1986, gives an example of the difference between duration hedging and our three-factor approach. Although this position was initially duration-matched, it

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Table 1 ▪ Illustrative Portfolio

Face Value ($)	Coupon	Maturity	Price on Feb. 5, 1986	Price on Mar. 5, 1986
-118,000,000	12 3/8	8/15/87	106 5/32	106 9/32
100,000,000	11 5/8	1/15/92	112 12/32	115 20/32
-32,700,000	13 3/8	8/15/01	130 16/32	141 14/32

would have lost $650,000 during this period.

The portfolio was especially sensitive to the factor we call curvature. We return to this example later, after we develop the statistical and analytical bases for our multifactor models, present empirical results explaining the returns on zeroes, and extend the analysis to more complex securities.

THE IMPLIED ZERO CURVE

In conducting a statistical study of the common factors that affect bond returns, we encounter an immediate difficulty: The relationship among returns on coupon bonds of distinct maturities changes over time as a function of the level of interest rates. When interest rates are high, returns on long bonds will be much closer to returns on shorter ones than in cases when low interest rates prevail.

We can address this problem by considering the basic building blocks of each coupon bond: the dollar payments at each maturity. It is among the returns to such dollar payments that we search for the common factors. To conduct statistical analyses of the returns to these *spot* obligations, we must derive the implied prices of dollar payments at different future maturities from coupon bond prices.

If we know the prices of zero-coupon bonds of all maturities, in principle we can derive the price of a given coupon bond by simply taking the sum of the coupon payments for each future date multiplied by the price of the corresponding zero-coupon bond. In reality, however, such an exercise would lead to a mispricing of coupon bonds, because of the peculiarities of the market for Treasury zeroes. Further, the amount of mispricing would change over time in response to changing conditions in the market for Treasury zeroes.

Ideally, what we need are "zero-coupon obligations" priced in such a way that they can be used to obtain, in the manner described above, the correct prices for coupon bonds. As such obligations do not exist, we are forced to find the prices of such securities that are *implied* in the prices of coupon bonds. In other words, we find the prices of zero-coupon instruments for each date that best explain the observed prices of coupon bonds. These *fitted* prices are the ones we use in the analysis that follows. We sometimes refer to the associated yields as *fitted yields.*

DURATION HEDGING

Financial analysts use the concept of duration to characterize the price sensitivity of a bond to a parallel shift in the yield curve. It is well understood that, in reality, yields *do not* always move in a parallel fashion. Hence, no matter how we define an average change of yields, such a parallel shift only partly explains the price changes at any time.

We may decompose the change in the price of a zero over any time period into two parts. The first part results from the aging of the bond over that period, while the second part is attributable to price changes in zeroes of constant maturity. The first part is not subject to uncertainty; it is the second part that interests us here.

Think of this part either as the return on a dollar's worth of a constant-maturity zero, or the change in the value of a zero caused by an instantaneous change in yields (as by choosing a small enough time period, we can make the portion of returns caused by aging arbitrarily small). We use the word "return" in these equivalent senses when we apply it to zeroes.

The return on a dollar's worth of a zero approximately equals the negative of the change in the zero's yield multiplied by its maturity. If we choose a particular bond as the *reference bond*, then we may express the yield of a zero as the sum of the yield of this reference bond plus the *spread* of the zero's yield off the reference bond. This allows us to express the return on a dollar's worth of a zero as the sum of two terms: 1) the product of the zero's maturity and the (negative of the) change in the yield of the reference bond, and 2) the product of the zero's maturity and the (negative of the) change in the spread off the reference bond.

For example, we may take the five-year zero as the reference bond and consider the return on a dollar's worth of the ten-year zero. Let us assume that initially the (continuously compounded) yield on the five-year zero is 10%, and the five to ten spread is thirty basis points. Suppose that the yield on the five-year zero subsequently increases to 11%, while the five to ten spread widens to fifty b.p. Then we can approximate the change in the value of a dollar's worth of the ten-year zero by:

$$\begin{aligned} & -(10 \times 1\%) - [10 \times (50 \text{ bp} - 30 \text{ bp})] \\ = \; & -(10 \times 0.01) - (10 \times 0.002) \\ = \; & -\$0.12\,. \end{aligned}$$

The first term is the product of the maturity of the bond and the change in the yield of the reference bond.

The second term is the product of the maturity and the change in spread (twenty b.p.). In this scenario, each dollar's worth of the ten-year would lose approximately 12 cents.

This decomposition suggests that, in the absence of spread changes, the effect of a yield change on the prices of zeroes of different maturities will be proportional to the maturity — which in this case is also the *duration* — of the bond.

Consider now a portfolio consisting of several bonds. The change in the value of this portfolio equals the sum of each security's return weighted by the dollar amount of the security in the portfolio. If all spread changes are zero, such a change will simply be proportional to the (dollar-value-weighted) duration of the portfolio. A duration-matched (zero-duration) portfolio will be immunized against the risk of yield changes.

If a portfolio has zero duration, the change in the value of the portfolio will simply reflect the changes in the yield spreads off the reference bond. As we argued above, each such change in yield spread of a zero affects the return on that zero in proportion to its maturity. In turn, this return affects the value of the portfolio in proportion to the dollar amount of that security in the portfolio.

Hence, to find the per dollar return on a portfolio with non-zero value, we take the negative of the sum, across all securities in the portfolio, of each security's maturity-weighted spread change (change in spread of security x maturity of security), multiplied by the dollar share of the security in the portfolio.

To illustrate this, let us now consider a portfolio that is long \$100 million (market value) of the ten-year zero and short \$50 million of the twenty-year zero. We again take the five-year bond as the reference bond. We assume that, initially, the yield on the five-year is 10%, the five to ten spread is thirty b.p., and the five to twenty spread is sixty b.p. As before, we consider the case in which the yield on the five-year moves to 11% and the five to ten spread to fifty b.p., and we suppose further that the five to twenty spread narrows to fifty-five b.p.

Recall that above we calculated the return on a dollar's worth of the ten-year as minus 12 cents. To compute the return on a dollar's worth of the twenty-year zero, we proceed exactly as before, to obtain:

$$
\begin{aligned}
& -(20 \times 1\%) - [20 \times (55\text{ bp} - 60\text{ bp})] \\
= \; & -(20 \times 0.01) - (20 \times -0.0005) \\
= \; & -\$0.19\ .
\end{aligned}
$$

Notice that the impact of the reference bond's yield change on the twenty-year bond is exactly twice its impact on the ten-year. Given the portfolio weights, the total impact of this yield change on the portfolio value is zero. By contrast, the spread changes have a net effect per dollar's worth of the portfolio equal to:

$$
\begin{aligned}
& -2/3\ [10 \times (50\text{ bp} - 30\text{ bp})] + \\
& 1/3\ [20 \times (55\text{ bp} - 60\text{ bp})] \\
= \; & -2/3\ (10 \times 0.002) + 1/3\ (20 \times -0.0005) \\
= \; & -\$0.017\ .
\end{aligned}
$$

Suppose now that the changes in each security's spread are independent both of the changes in any other zero's spread and of the changes in the yield of the reference bond. Although duration hedging no longer eliminates all risk, it immunizes the portfolio against a change in the yield of the reference bond. In other words, duration risk eliminates all *systematic* risk. We can diminish the leftover uncertainty through diversification.

To see this, notice that because the changes in spreads are independent of each other, the variance of the return on a zero-duration portfolio equals the sum, across all securities in the portfolio, of each security's weighted spread change variance (variance of spread change × *square* of duration × *square* of dollar share of the security in the portfolio). By increasing the number of instruments within the same maturity spectrum, we can make the dollar share of each security arbitrarily small, and hence make the variance of the *return* per dollar invested in the portfolio as small as we want.

MULTIFACTOR MODELS

In the case just discussed — where the changes in each security's spread are independent both of the changes in any other zero's spread and of the changes in the yield of the reference bond — we can express the return on each security in the portfolio as the sum of two components. The first is proportional to the *yield change of a reference bond*, and the second represents the *asset's own movement*. The change in the yield of the reference bond is the common factor that affects all returns. The second term associated with each zero is the *specific factor*, or *error*.

Each instrument has its own sensitivity to the common factor. For instance, the sensitivity of a zero to the common factor — i.e., to the yield change of the reference bond — is exactly its maturity. If this were a complete description of the real world, then duration hedging would eliminate all systematic risk. In reality, however, other systematic risks are present in the market — that is, there are other common factors that affect bond returns. In fact, our research indicates that there are *three* major sources of aggregate risk.

We refer to the sensitivity of a bond's returns to a common factor as the *loading* of the bond on that factor. We can determine the sensitivity of a *portfolio* to that factor simply by adding up each asset's loading on that factor weighted by the asset's share in the value of the portfolio. Moreover, we can form portfolios that are immune to a factor by choosing asset holdings that make this weighted sum equal to zero. Notice that to hedge correctly all you need to know are the loadings.

Although in the examples above the single common factor is *observable*, you could do as well with *unobservable* factors, provided that you could still identify the loadings. What is more important, the correct model may involve unobservable factors. For instance, it is widely believed that changes in Federal Reserve policy are a major source of changes in the shape of the yield curve. If so — even though we have no clear idea of how to measure "policy" — we can insulate ourselves against Fed policy changes provided only that we can determine the *relative* effects of these changes on the returns to bonds of different maturities.

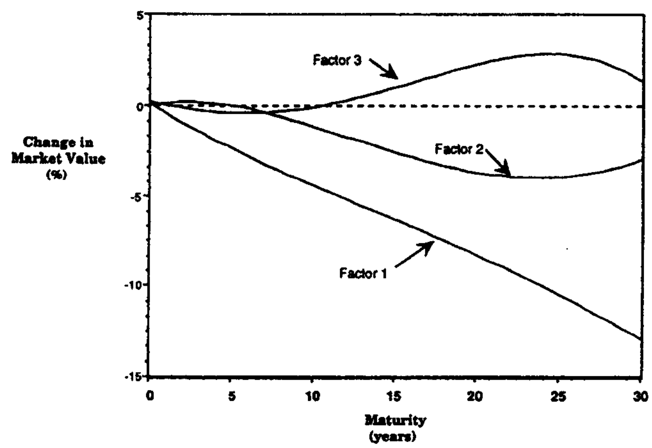
When dealing with unobservable factors, we may assume for convenience that each factor has a mean of zero and a unit variance, and that the covariance between any two distinct factors is zero. If we have posited m common factors, we can decompose the variance of the returns of any security into the sum of $m + 1$ terms. One term is the variance of the security's specific factor, and each of the other terms is the square of the security's loading on a common factor. We can use this decomposition to determine the proportion of the variance of the return on a particular security explained by each factor.

A THREE-FACTOR MODEL FOR TREASURY BONDS

We are now ready to examine a three-factor model of the Treasury bond market. The model we develop here and in the next section deals with zero-coupon instruments. In the section following we illustrate application to coupon bonds.

Although the cases we have dealt with so far have focused on determination of the returns to securities, there are other natural candidates for the role of dependent variable. One possibility is yield changes. A better choice, however, is *excess returns over a risk-free rate*. Not only do good

FIGURE 1 ▪ Factor Loadings



theoretical reasons exist to choose excess returns as the dependent variable, but empirical evidence also supports the choice of excess returns where the generic overnight repo rate is used as the risk-free rate.[1]

Examination of the data indicates that a three-factor model fits quite well.[2] First, a likelihood ratio test shows no evidence against the three-factor model. Second, the three-factor model explains — at a minimum — 96% of the variability of excess returns of any zero.

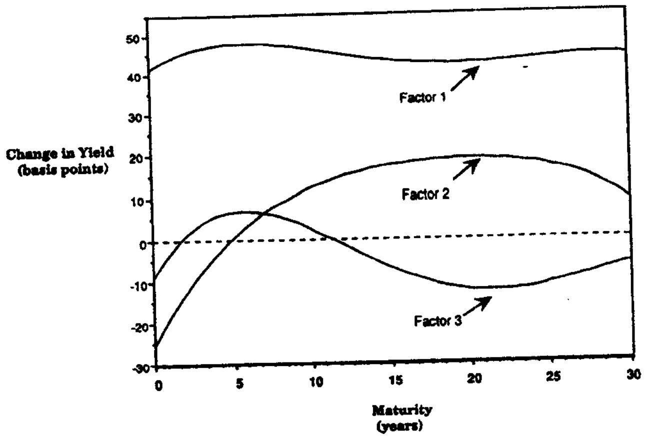
We display the factor loadings of the zeroes in Figure 1.[3] In Figure 2 we show the impact of the factors on the *yields* of the zeroes. The curve for each factor represents the change in yield caused by a shock from that factor of one standard deviation (so that all the shocks are equally likely events).

An examination of Figure 2 shows that the yield changes caused by the first factor are basically constant across maturities. That is, the first factor represents essentially a parallel change in yields, although there are some differences, especially in the two- to seven-year maturity range. Thus, hedging against Factor 1 is close to duration hedging, although it requires slightly less of a five-year bond to hedge against a long bond than would duration hedging.

Note that our method of identifying factors implies that a movement in the yield curve similar to the one caused by Factor 1 is more likely to occur than a roughly parallel shift. The impact of Factor 1 on yield levels leads us to name it the *level* factor.

We call the second factor *steepness*, even though it does not correspond exactly to any of the steepness measures commonly used. Figure 2 shows that a shock from the

FIGURE 2 ▪ Yield Curve Impact



steepness factor (as defined here) lowers the yields of zeroes up to five years, and raises the yields for zeroes of longer maturities. Notice that the impact on yields peaks at around eighteen years and is in fact smaller at the long end of the curve.

The third factor, which we call *curvature*, increases the curvature of the yield curve in the range of maturities below twenty years; the effect on yields tails off above twenty years. In an earlier paper co-authored with Laurence Weiss,[4] we examined the impact of changes in interest rate volatility on the shape of the yield curve and discovered the same pattern. We found that changes in curvature of the yield curve are associated with changes in rate volatility.

Table 2 uses the variance decomposition discussed in the previous section to show the relative importance of the three factors. As you can see, the first factor is by far the most important; for the entire set of zeroes, it accounts for 89.5% of the total explained variance. Factor 2 is responsible

Table 2 ▪ Implied Zeroes: Relative Importance of Factors ▪ (*Percent*)

Maturity	Total Variance Explained	Proportion of Total Explained Variance Accounted for by: Factor 1	Factor 2	Factor 3
6 months	99.5	79.5	17.2	3.3
1 year	99.4	89.7	10.1	0.2
2 years	98.2	93.4	2.4	4.2
5 years	98.8	98.2	1.1	0.7
8 years	98.7	95.4	4.6	0.0
10 years	98.8	92.9	6.9	0.2
14 years	98.4	86.2	11.5	2.2
18 years	95.3	80.5	14.3	5.2
Average	**98.4**	**89.5**	**8.5**	**2.0**

for an average of 81% of the remaining variation in returns. The table supports the idea that "first factor" hedging — or its close cousin, duration hedging — takes care of most of the return risk. Nonetheless, even a duration-hedged portfolio can be subject to substantial losses in particular market episodes, as we will show below.

MIMICKING PORTFOLIOS

We discussed above how to construct portfolios that were *hedged against* a particular factor. Although our factors are unobservable, by using enough securities we can construct portfolios that are sensitive only to the movements of a particular factor — i.e., portfolios that *mimic* a factor. In our three-factor model, constructing a portfolio that is sensitive to only a single factor involves solving three equations with n unknowns, which is generally possible when n, the number of securities in the portfolio, is greater than or equal to three.

When the number of securities exceeds three, there are an infinite number of portfolios that mimic a given factor, but we can easily narrow the choice. Besides its sensitivity to a single common factor, the return on a portfolio will depend on the variance of the specific factor of each security, weighted by the (square of the) share of that security in the portfolio. For a given dollar value, the portfolio that minimizes this "specific" variance is the ideal mimicking portfolio.[5]

We can use mimicking portfolios to gain further insights into the factors. As we indicated earlier, the impact of the third (curvature) factor on bond yields is similar to the predicted impact of volatility on yields.[6] To test empirically the relationship of the third factor to volatility, we constructed a portfolio to mimic volatility and correlated the returns on this portfolio to those of our factor-mimicking portfolios.

To do this, we start with the price volatility implicit in options on Treasury bond futures — a measure of volatility on which many market participants rely. We divide the implied price volatility by the duration of the cheapest bond to deliver, to derive an implied yield volatility. Then — as suggested by our earlier study — we regress this variable on the yields of the one-month, three-year, and ten-year zeroes.

The returns to a portfolio where each of these three instruments is present — in a proportion equal to its weight in the regression divided by its duration — can thus be taken as mimicking the volatility implicit in the yield curve. The correlations of the monthly returns on this portfolio with those of each of the three mimicking factor portfolios were 0 for Factor 1, 0.02 for Factor 2, and 0.90 for Factor 3. Clearly, the returns on the "volatility portfolio" are highly correlated with the returns on the "curvature portfolio."

This relationship becomes important when we treat securities that contain embedded options; such securities are naturally sensitive to volatility.

FACTOR SENSITIVITIES OF OTHER SECURITIES

We can easily extend the factor model to deal with securities other than zeroes by modeling the prices of these securities as a function of the prices of zeroes. We illustrate this procedure with three types of instruments: non-callable coupon bonds, callable Treasuries, and long-bond futures.

Non-Callable Coupon Bonds

Suppose a coupon bond has two payments, with each payment amount and date coinciding with the face value and maturity date of a zero. The return on the coupon bond is the weighted average of the returns on each of the zeroes, where the weight of a particular zero is the fraction of the bond price attributable to that zero. Thus, the factor sensitivity of the return on this bond equals the same weighted average as the factor loading of the zeroes. Note that even though the factor sensitivities of zeroes are constant over time, the factor sensitivities of coupon bonds will vary with market levels.

To illustrate how much of the variation in coupon bond returns is explained by the factor approach, we examined bond returns of some liquid Treasury issues. For the observations on prices, we used weekly Wednesday prices from February 22, 1984, through August 17, 1988, taken from Goldman Sachs 3 p.m. quotes. Some of the bonds were issued during the sample. Table 3 displays the results.

The excess return explained by the three-factor approach is the sum of the factor loadings for each bond times the returns of the mimicking portfolios. As you can see in the third column in Table 3, the three-factor approach explains no less than 94% of the total variance of returns. On average, it explains about 97%.

To compare the factor approach with the traditional duration approach to hedging, we also show in Table 3 the percent of variance of each bond not explained by factor hedging (column 4), duration hedging with one bond (column 5), and duration hedging with a portfolio of bonds (column 6). Column 4 is equal to 100 minus column 3.

Column 5 shows that duration hedging one bond with another bond can work quite well, or quite badly, depending on how similar the hedge bond is to the bond being hedged.[7] Column 6 shows the more relevant comparison of duration hedging to the factor approach. Here, each bond was duration-hedged against a portfolio consisting of all of the bonds from Table 3 outstanding at each date. The weights used were inversely proportional to the durations of the bonds, so that each bond made an approximately equal contribution to the total return of the portfolio.

Relative to the duration hedge, the three-factor hedge reduces the residual variance by an average of 28%. Of course, this is just the average reduction of variance. As we show below in an example, the improvement in hedg-

Table 3 ▪ Explanation of Returns on Treasury Coupon Bonds*

Maturity	Coupon (%)	% of Variance of Excess Returns *Explained* by the Three Factors	% of Variance of Excess Returns *Unexplained* by the Three Factors	Duration Hedging With One Bond	Duration Hedging With a Portfolio of Bonds
05/15/89	11.75	96.6	3.4	16.8	8.0
04/15/91	12.375	97.2	2.8	11.7	4.4
07/15/92	10.375	97.6	2.4	10.3	3.5
02/15/93	7.875	95.6	4.4	11.0	4.3
02/15/96	8.875	98.2	1.8	6.6	2.2
02/15/01	11.75	96.7	3.3	3.3	3.5
02/15/03	10.75	97.6	2.4	1.0	2.8
11/15/03	11.875	97.9	2.1	2.1	3.3
08/15/05	10.75	98.7	1.3	0.7	2.4
02/15/06	9.375	98.5	1.5	0.0	2.8
11/15/15	9.875	97.8	2.2	2.5	3.2
02/15/16	9.25	94.2	5.8	7.2	7.0
05/15/16	7.25	97.1	2.9	4.1	4.2

*Weekly returns from Feb. 22, 1984, through Aug. 17, 1988.

ing through the use of three factors may be much greater in particular instances.

To find the factor sensitivities of a portfolio of bonds, we simply add up the loadings of each component bond weighted by the bond's market value. We can apply this method to the illustrative portfolio described in Table 1. We find that the portfolio's sensitivities (in thousands of dollars) to a one-standard-deviation (monthly) shock in each of the factors are:

Sensitivity to Factor 1	- 84.01
Sensitivity to Factor 2	6.50
Sensitivity to Factor 3	-444.95

As we suggest at the outset, this combination is highly sensitive to the curvature factor but is much less responsive to the other two factors. Notice that duration hedging eliminates most, but not all, of the Factor 1 risk.

Callable Treasuries

We must use an option pricing model to be able to treat this bond as a non-callable host minus a call option. We can derive the factor sensitivities of the non-callable host as above. We then use the option-pricing model to find the sensitivity of the call option to each of the factors. Remember that the option value is also sensitive to changes in volatility and that, as we noted, changes in volatility are highly correlated with changes in curvature.

Long-Bond Futures

To find the impact of factor changes on futures prices, we use a model for pricing futures as a function of volatility and bond prices. Using the mimicking portfolios, we then construct a measure of the proportion of the futures price change explained by the factor changes. Using weekly data from January 1, 1986, through June 14, 1988, we find that for the nearest futures contract this measure has a correlation of 0.97 with the changes in futures prices.

APPLYING THE FACTOR APPROACH

We can now complete — in terms of our factor model — the analysis of the position described at the beginning of the article. Using the mimicking portfolios, we can determine that between February 5, 1986, and March 5, 1986, there were these movements in the three factors, in multiples of one standard deviation:

Level -1.482 **Steepness** -2.487 **Curvature** 1.453

Rates came down severely, and the yield curve became markedly more curved and flatter. The rate-level change, however, had only a small effect on the position, as the sensitivity to the first factor was just -\$84,010 per one-standard-deviation change. Hence, the level change caused a *gain* of (-\$84,010 × -1.482) = \$124,503. The curvature change, however, was responsible for a loss of (\$444,950 × 1.453) = \$646,512. In total, the three factor shocks explain a loss in the value of the portfolio of \$538,200.

The cost of holding a portfolio over a given period has two elements. The first is the cost of holding the position if there is no change in the yield curve. Its actual value depends, in particular, on the assumed repo and reverse repo rates. The second is that part caused by the yield curve movements.

Using the one-day generic repo rates that prevailed during the period, we estimate that the total loss of the position was \$676,200. Of this, the part corresponding to the yield curve movements was \$658,000, because in reality the position would have lost \$18,200 if the yield curve had remained constant.

The three factor movements explain all but \$119,800 of the loss caused by the yield curve changes. This error is partly explained by the fact that as the yield curve moved during the period, the factor loadings of the coupon bonds also changed (the remainder is due, of course, to the specific factors of each security). Nonetheless, the initial factor loadings explain 82% of the variation.

The instance we chose to illustrate our hedging was particularly severe. Such a choice is not unwarranted, however, for the main justification for hedging is precisely the fact that such episodes, although rare, *do* occur. In fact, the result would have been even worse had we not made the portfolio insensitive to steepness changes. That is, a duration-hedged portfolio sensitive to steepness would have suffered even bigger losses (or enjoyed bigger gains!).[8] Of course, to have hedged against the three factors, we would have had to use at least four bonds.

CONCLUSION

The results of our investigation strongly suggest that there are three principal common influences on the variation in bond returns represented by the zero yield curve. We also derived the theoretical impact of these same influences on more complicated Treasury securities. Empirical estimates indicate that these factors explain most of the returns on the securities we examined.

Although we have focused on bond portfolio hedging, we can apply the methodology to any asset or liability stream for which we can derive a covariance with bond returns.

ENDNOTES

[1]We confirmed this by examining the alternative covariance matrixes to test the hypothesis that they had remained unchanged over our sample period — which is implicit in factor models.

[2]We fitted the model to weekly observations from January 1984 through June 1988.

[3]Because we use unobservable factors, we must contend with a natural ambiguity: We can create a new set of factors by using combinations of the initial set of factors. We deal with this ambiguity here by choosing the first factor such that we maximize the proportion of variance explained. We then choose the second factor, among those that have zero covariance with the first one, to maximize the proportion of the remaining variance explained, and so on. From the viewpoint of hedging, it should be clear that once we are hedged against *all* factors in a model, we are hedged against all factors in any model produced by combinations of the initial set of factors.

[4]Robert Litterman, José Scheinkman, and Laurence Weiss, "Volatility and the Yield Curve," Goldman, Sachs & Co., August 1988.

[5]We can construct such a portfolio by solving a quadratic minimization problem subject to linear constraints.

[6]We developed the reasoning behind this in our earlier paper. See Litterman, Scheinkman, and Weiss, op. cit.

[7]In our example the hedge bond until February 12, 1986, was the 10.75% coupon bond of 2/15/03. After February 12, 1986, we used the 9.375% coupon bond of 2/15/06, which was issued at that time.

[8]Our portfolio still exhibited non-zero convexity. But eliminating the convexity risk would not have greatly improved the situation. To be simultaneously duration- and convexity-hedged, a portfolio in which the investor was long exactly the same amount as before of the 1992 bond ($100 million) would require being short $204 million of the 1987 bond and $19,000 of the 2001 issue. Such a portfolio would have gained $525,400. But from the viewpoint of hedging, this outcome is as bad as losing.