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Close-Out Risk Evaluation (CORE): A New Risk-Management Approach for Central Counterparties

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Abstract

This paper introduces the CORE methodology for managing risk of multi-market central-counterparties. CORE generalizes the classical SPAN method of stress scenarios by incorporating explicitly market liquidity of listed instruments and modeling the liquidity profile of OTC instruments and the liquidation-by-auction mechanism. In the presence of liquidity constraints, there is a difference between *ex ante* mark-to-market losses in

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case of stress moves and realized losses after liquidation of a portfolio is terminated, which leads us to formulate an optimal liquidation strategy for any portfolio under stress-scenarios and liquidity constraints. We formulate the problem as the maximization of an objective function which involves potential transient losses (realized and mark-to-market). The objective function is designed to reduce funding liquidity requirements for the CCP along the liquidation period and is shown to be robust with respect to liquidation-by-auction assumptions and liquidity constraints. Mathematically, we formulate the CORE problem as a Linear Programming problem under convex constraints, which can be solved using high-performance large-scale optimization packages such as CPLEX and GUROBI. Finally, we illustrate the computation of the CORE strategy on a few examples of portfolios and compare the anticipated gains in reserve capital for the different examples. Preliminary tests indicate that improvements of between 20 and 60 % can be realized by applying optimal liquidation schedules which match instruments with common risk factors according to their liquidity profiles in the liquidation process.

Contents

1 Introduction

Since its introduction in the early 1990's, Value at Risk (VaR) has been the main framework for the risk-management and supervision of financial institutions[1]. Yet, VaR has been widely criticized for its inability to quantify extreme losses beyond a probability percentile, as well as its inability to capture liquidity risk. Another approach to risk-management is to subject portfolios to extreme market scenarios. The idea of evaluating risk via stress-scenarios is particularly appropriate for portfolios of derivatives or instruments with embedded leverage, and has been put in place by the major derivatives exchanges and clearinghouses[5].

The scenario-based approach is often referred to as SPAN.[1] The problem with SPAN as it stands is that it does not take into account explicitly the liquidity of the instruments in the portfolio. In other words, it quantifies potential unrealized, or *mark-to-market* (MTM) losses, but does not anticipate the cost of unwinding a portfolio. Due to liquidity constraints, the latter is expected to be more severe than MTM losses prior to liquidation.

This paper discusses a new approach for managing financial risk – one which combines the stress-scenario analysis with liquidity risk-analysis. The new framework is based on the Close-out Risk Evaluation (CORE) proposal put forth by BM&FBovespa (BVMF) [2],[3]; the contribution of this paper can be viewed as a practical implementation of CORE based on linear programming. See [4] for a shorter, more technical description of the Finance Concepts implementation of the CORE algorithm.

The motivation for extending VaR and SPAN is to create a unified framework for clearing heterogeneous portfolios consisting of multiple underlying assets having exposure to common risk factors (*e.g.* equities, equity indices, currencies in the same geographic zone), as well as instruments with very different liquidity profiles and trading venues (*e.g.* OTC swaps versus futures). With the ultimate goal of creating a unified risk management approach for a multi-asset, multi-market clearinghouse, CORE aims at including explicitly the liquidity (and liquidation) characteristics of a large universe of instruments and model their risk exposure to common factors.

Following [2] [3], [4], our approach is based on (i) implementing stress-scenarios relative to common risk factors and (ii) taking into account the (limited) market liquidity of each instrument. In particular, the CORE approach gives rise naturally to a *liquidation time period* and thus the choice of a *close-out*

[1] SPAN is actually the acronym for the CME risk system, and stands for Standard Portfolio Analysis of Risk.

strategy for any given portfolio. Schematically, CORE views the portfolio risk-management problem as a optimal liquidation problem, relying on SPAN-like measures to evaluate risk during the liquidation period. The risk of a portfolio to a clearinghouse is equated with the *ex ante* potential cost of liquidating it under adverse conditions.

As we shall see, an appropriate choice of *liquidation strategy* for a portfolio is crucial. A poor choice may give rise to a unwarranted losses over the liquidation period due to market fluctuations, and require high risk requirements; a good choice could take advantage of risk-offsets between instruments which are liquidated simultaneously, requiring less capital requirements.

The *absence of market liquidity* represents the impossibility to transact in a security due the market's inability to clear the trade at a known price over a specific period of time. From a practical point of view, market liquidity in a given security is often modeled as the ability to transact a fraction of its average trading volume over a time period. The other important notion is that of *funding liquidity.* The lack of funding liquidity for an agent means that his liabilities may not be met when they are due, or can be met but only at a large financial cost. Both aspects of liquidity risk (market liquidity and funding liquidity) are present when stress-testing a bank, a non-bank financial institution or a portfolio.[2]

Liquidity risk is particulary important in the case of central counter-parties (CCPs), such as the clearinghouses run by exchanges.[3] The function of a CCP is to take the opposite side of every transaction through trade novation, elim-

[2]Arguably, many financial debacles of the last 20 years may be attributed to the failure to anticipate liquidity problems. Well-known examples of firms which collapsed due to lack of funding liquidity include Metallgeselschaft AG (corporate), Long-Term Capital Management (hedge fund), Bear Stearns Cos. (broker-dealer), Lehman Brothers (broker dealer), MF Global (broker-dealer), and many others. Bank runs can be viewed as funding liquidity crises.

[3]Some examples of CCPs include: the Depository Trust and Clearing Corporation (DTCC), the Fixed-Income Clearing Corporation (FICC) the Options Clearing Corporation (OCC),the Chicago Mercantile Exchange (CME) Clearing, ICE Clearing, Eurex Clearing, CBLC, SETIP, *etc.*

inating counterparty risk between participants. In the case of exchanges, in which price information is public, central-clearing is an efficient mechanism for trading, pre- and post-trade analysis, as well as risk management. Clearinghouses for over-the-counter (OTC) trading, such as FICC (government bonds and mortgage-backed securities) and ICE Clearing (credit-default swaps) play a crucial role in the well-functioning of fixed-income and credit markets.[4] In the latter case, centralized clearing also converts bilateral risk into central counterparty risk. [5]

Let us examine counterparty risk from a CCP's viewpoint. What happens if one or several clearing participants default? To answer this questions, we note that there are mainly two models of CCP risk systems in terms of loss attribution: (i) the *defaulter pays* model and (ii) *loss mutualization* model. In the defaulter pays model, which has been adopted by most exchanges, if a participant defaults the CCP liquidates his positions and collateral. If the result is a credit, the balance is returned to the defaulter. If it is a loss, the CCP will absorb it using its own funds or by tapping into its credit lines. In the case of loss mutualization, the CCP has an additional source of funds, often called a *guarantee fund* (GF). If a participant defaults, the CCP will liquidate his portfolio and, in the case of a debit, use the GF to meet any remaining liabilities. Nevertheless, there exists a remote possibility that the guarantee fund will be exhausted, in which case the losses should be covered by CCPs capital and credit lines.[6] In either case, the risk to the CCP is to have to liquidate the position of the participant under adverse conditions.

[4]The Dodd-Frank Act of 2010 requires that a large class of over-the-counter derivatives, including interest rate swaps, be centrally cleared.

[5]Another advantage of central clearing of OTC trades is that it has *regulatory transparency*, *i.e.* regulators can monitor the OTC market and assess the potential for systemic risk, as well as contributing to "trade certainty". In some jurisdictions, central clearing of OTC products is required by law but in many other venues it remains a voluntary option for market participants.

[6]Mutualization is used mostly by OTC clearinghouses such as ICE Clearing, whereas exchange CCPs implement mostly the defaulter pays model. The reason for this is structural, as capital requirements for traders in listed markets and OTC markets are generally different.

To understand the CCP's risk exposure, we need to estimate the market liquidity of each component of the portfolio. Some assets may be relatively liquid; others less. For OTC instruments, the CCP will typically organize an auction among clearing participants. It is natural to expect that, in some cases, OTC instruments will not be able to be liquidated for several weeks. Listed instruments will be more liquid and can be closed earlier.

Some portfolios have natural offsets between instruments of different liquidity; for instance, on-the-run and special government bonds may have the same exposure to the term-structure of interest rates, but very different market liquidity. Thus, the question from the CCP's perspective is how to organize and schedule the liquidation of portfolio to mitigate risk. Liquidating all instruments in the portfolio as soon as possible may produce larger capital requirements for the CCP than doing a more "orderly" close-out.

Example. To better understand the problem at hand, we consider an example in which a participant's portolio includes liquid exchange-traded instruments and OTC forwards on the same underlying asset. In the case of OTC contracts, the CCP must contact participants in order to liquidate the OTC position bilaterally or, alternatively, it must organize an auction for this purpose. Assume that the portfolio consists of two positions: an OTC forward contract to deliver 10,000,000 shares of BOVA11 in 1 year (the ETF tracking the IBOVESPA index), settled in cash, and a long position in 10,000,000 BOVA11. The exchange believes that it can liquidate the forward contract in an auction in 2 weeks ($t = 15$). Suppose that the market liquidity in the listed ETF is such that 10,000,000 shares can be absorbed by the market in a single day (so there is negligible market liquidity risk).

Consider the following two strategies for unwinding the position:

1. the CCP will sell the 10,000,000 ETFs on $t = 1$ and then auction the

forward contract in 15 days;

2. the CCP will sell the 10,000,000 ETFs on $t = 15$ and auction the contract on $t = 15$ as well.

Let us calculate the charges (or *anticipated funding needs*) that might apply in each case.[7] We assume (arbitrarily) that the capital charge due to market risk for 10,000,000 BOVA11 is 100 million BRL.

We note that, since the portfolio is BRL-neutral the potential MTM loss (unrealized loss) on $t = 1$ is nil.[8]

In close-out strategy 1, if we plan to liquidate the 10MM BOVA11 on date $t = 1$, we might think that we need to charge 100 MM due to the fact that the ETF could be liquidated at a large loss on that day. However, such loss will be offset by a mark-to-market gain in the forward contract, so no risk charges apply on $t = 1$. On $t = 2$, however, we still could have a potential realized loss of 100 MM (say) from the liquidation of the ETF shares on the day before; and the fact that we still hold open the forward contract on $t = 2$. If we follow the same "charges" approach, this means that we now require 100 MM to cover the potential loss on $t = 1$ and *another* 100 MM dollars to cover against potential future on the forward contract (mark-to-market loss). The same risk exists on $t = 3, ...,$ up to $t = 15$. The conclusion is that the CCP will require BRL 200 MM during 14 days to cover realized and unrealized potential losses during the liquidation period.

In contrast, in close-out strategy 2, the CCP chooses to keep the entire portfolio on the books until $t = 15$. Clearly, there is no risk of any intermediate realized losses, since there is no intermediate share sale: each variation in

[7]By capital charges we mean the variation margin, or cash guarantee that must be deposited in the participant's account to safeguard against potential shortfalls.

[8]The initial MTM loss is independent of the liquidation strategy. It can be viewed as the *ex ante* worst loss if all positions could be liquidated on date $t = 1$ regardless of liquidity considerations.

the share price is offset by a MTM variation in the forward. The total MTM charges for the short-forward/long-cash position is essentially zero, and we do not anticipate in this case any funding shortfalls.

The conclusion is that the portfolio close-out strategy is important. In the first case, the CCP would have to reserve 200 MM in margin or risk eventually to have to use its cash reserves or lines of credit. In the second case, there is no need for a risk charge, since the position is essentially hedged until the end of the close-out period.(Notice that we assume no basis risk between the forward and the cash, for simplicity.) It also shows that the naive strategy of closing out all positions as soon as possible according to their liquidity is suboptimal. In this example, due to the fact that the ETF is assumed to have large liquidity, the effective anticipated cost of the strategy is equal to the MTM loss on day $t = 1$ (zero).

Next, we consider a variant of the above example, in which market liquidity is reduced. Assume that the market liquidity of BOVA11 is such that only 5,000,000 shares (50% of the total position) can be liquidated on any given day, including the auction date. In particular, the CCP will not be able offset completely the risk of holding the forward contract until $t = 15$. In this case, there are (at least) two possible close-out strategies:

1. sell 5,000,000 ETFs on $t = 1$, sell 5,000,000 on $t = 15$, and auction the forward on $t = 15$ (early close-out of the 5MM ETF 'stub')

2. sell 5,000,000 ETFs on T+14, sell 5,000,000 on $t = 15$, and auction the forward on $t = 15$ (late close-out of the 5MM ETF 'stub')

Notice that the MTM loss on day $t = 1$ is zero, since the position is exactly market-neutral. In close-out strategy 1, we should require a BRL 50 MM charge for the liquidation risk on $t = 1$, but this is offset by MTM gains on the remaining position – no charges are needed on $t = 1$. Starting on $t = 2$ there is a charge of

50MM against potential realized losses on the market sale on $t = 1$. In addition to this, we need to anticipate another 50MM for MTM losses from $t = 2$ to $t = 15$. The total funding needs are (i) 0 dollars on $t = 1$, (ii) 100M from $t = 2$ to $t = 15$. In close-out strategy 2, we do not expect to close out any position until $t = 14$. Therefore, we require (i) 0 BRL until $t = 13$ (ii) BRL 50 MM on $t = 14$ and (iii) BRL 100MM (50+50) on $t = 15$.[9]

The second example shows that in the absence of a perfect hedge due to reduced liquidity, we have different possibilities in terms of close-out strategies. Both have the same worst-case capital requirement (BRL 100MM), but there is an important difference: the first strategy requires reserving 100 MM dollars for 14 days. The second strategy requires an ex-ante charge of 50MM dollars for 13 days and 100 MM dollars for only one day. If we accept the premise that the CPP's risk is funding liquidity risk, the second strategy is preferable, even though both have the same worst-case capital requirement. Another strong argument for strategy 2 is the fact that this strategy would be much better in terms of funding requirements if the CPP could, in practice, terminate the OTC contract *before* $t = 15$, because the strategy would not realize unnecessary losses, keeping MTM losses as small as possible for the longest period of time. This argument is important since the process of liquidation of OTC products has an element of uncertainty. Thus, strategy 2 is consistent with viewing $t = 15$ as a worst-case liquidation date for the OTC forward.

Based on this example, we or any given a portfolio, the CCP should seek a close-out strategy which minimizes its potential losses over each day of the close-out period. Every close-out strategy gives rise to a sequence of requirements, say:

[9]Notice also that the naive strategy of liquidating the ETFs as fast as possible,in two days, and the forward at the end leads to a BRL 200 MM requirement after day 3, which is really suboptimal.

$$0 \geq L_1^* \ \geq L_2^* \geq .... \geq L_{T_{max}}^*,$$

which correspond to estimates of potential losses during the liquidation period under extreme scenarios. Here, requirements are represented as negative numbers, and T_{max} is the last date in the liquidation period. If we view these requirements as funding costs for the CCP, a reasonable close-out strategy for the CCP should be one that minimizes the worst loss

$$\min_{1 \leq t \leq T_{max}} L_t^*$$

or the sum of the losses

$$\sum_{t=1}^{T_{max}} L_t^*. \tag{1}$$

It is important to note that the sum penalizes the magnitudes of loss requirements at all dates, not just at the final date. If two close-out strategies give rise to the same maximal loss, maximizing the sum will give preference to the one which has smaller "intermediate" losses for $L_t^*, \ t = 1, 2, ..., T_{max} - 1$. This underscores the preference for minimizing the CCP's funding needs over the liquidation period, as well as providing robustness over the final "auction date". In the second example discussed above, both strategies have the same worst-case requirement (100 MM) but the second strategy has smaller requirements for most days in the liquidation period and would fare better if the auction date was shorter than 15 days.

Once the optimal close-out strategy has been determined, the CCP should protect itself by requiring appropriate margin in the form of cash or securities. For example, a possible requirement is that the initial value of the portfolio including collateral be higher than *largest one-day loss*, $|L^*_{T_{max}}|$.[10]

This is, in essence, a description of the Close Out Risk Evaluation (CORE) methodology, which will be described technically in the rest of the paper.

2 Portfolio pricing during the liquidation period

2.1 MTM functions and liquidity constraints

In this paper, a portfolio is represented as a list of financial instruments (exchange traded or OTC), along with the quantities $Q_1, Q_2, ..., Q_n, ..$ of each instrument or contract held in the portfolio. Long positions are denoted by a positive sign and short positions with a negative sign.

Our definition of portfolio includes also collateral posted as margin. The reason for this is that collateral may provide natural hedges for financial instruments in the liquidation process. For instance, we would like to treat cash equities posted as collateral as Deltas which can be used potentially to hedge option contracts. Also, the collateral posted and the actual instruments in the portfolio may have different market liquidity constraints.

Every instrument cleared by the CCP is associated with a pricing or mark-to-market function, which is used to calculate the value of the security under different scenarios. If we consider a portfolio with N instruments, each instrument is associated with an MTM function

$$MTM_i(t, \xi_t) \tag{2}$$

where $t = 1, 2, ..$ represents the number of days following the calculation date and ξ_t represents a vector of market variables necessary to calculate the price

[10]For example, in margin calculations, statistical weights might be applied to different market "paths". This paper is primarily concerned with close-out strategies; the calculation of margin requirements using CORE will be treated in a separate study.

of the instrument.

The mark-to-market value of the portfolio at time $t = 0$ is therefore given by

$$V_0 = \sum_{i=1}^{N} Q_i MTM_i(0, \xi_0). \tag{3}$$

In addition to a MTM function, each instrument is associated with a *daily market-liquidity constraint*. The liquidity constraint represents an upper bound on the number of contracts that can be traded – long or short – without impacting the price, or for which the pricing function remains valid. This constraint is taken to be taken to be a percentage of the daily trading volume, γV, where $0 < \gamma < 1$, *e.g.* $\gamma = 10\%$.

Thus a full specification of a portfolio includes a list of all instruments, their MTM functions and the corresponding market liquidity constraints. In the case of OTC instruments, the liquidation is assumed to take place for the full amount on a future auction date, say T_{max} days after the calculation date. For OTC instruments liquidity is assumed to be 100% on the auction date and zero on any preceding date. This simple approach allows for *ex ante* modeling of liquidity mis-matches between listed and OTC products.[11]

2.2 Risk factors and risk scenarios

Each cleared instrument is assigned a set of *risk factors* which are used to evaluate the MTM function under different scenarios.

Risk factors are divided into *spot risk factors*, *term-structure risk factors* and *option volatility risk factors*. For instance, an FX rate can be considered as a spot risk-factor, whereas the interest rate swap curve (PRE, CUPOM) is, by

[11] Both the market liquidity constraints for listed instruments, as well as the auction dates for OTC instruments should be interpreted in the sense of being conservative estimates of actual liquidities in each case.

definition, associated with a term structure. Option volatility risk-factors are generally modeled as either spot or term factors, depending on the particular context.[12] The usual market convention is that factors which are associated with term structures are actually given in the form of curves interpolated from implied values for instruments with different maturities.

A *risk scenario* R is defined as a change in the value of the risk factors which could take place on a date following the calculation date. In the case of spot risk factors, scenarios take the form of a range of uncertainty

$$x_{min}(t) \leq x \leq x_{max}(t), \quad t = 1, 2, 3, ...,$$

typically involving a discrete set of values for x in this range. It is useful to make the range $[x_{min}(t), x_{max}(t)]$ become wider in time, to model the fact that uncertainty increases with time and thus, all things equal, it preferable to close out a position as soon as possible.[13] For the case of term-structure factors, the idea is to generate a discrete set of "deformations" or future states of the term-structure, also with time-dilatation. We shall use the notation

$$\xi_t \;=\; \xi_t(R) = R_t,$$

to represent the future state of the variables ξ_t on a particular day corresponding to a particular risk scenario.

Example: The instrument is an OTC European Call option on USD/BRL maturing in 63 days. The pricing function will be a function of the current

[12] Option volatility markets are generally quoted as a surface in strike and time-to-maturity. For purposes of risk-management, there needs to be a careful analysis of the correlation between different term/strikes pairs to prevent spurious risk-offsets *e.g* calendar spreads for single-name equity options.

[13] One possible prescription to generate "expanding" scenarios is to set $x_{min}(t) = x_{min}(1) - c\sqrt{t-1}$, $x_{max}(t) = x_{max}(1) + c\sqrt{t-1}$, where c is an appropriate coefficient linked to the daily volatility of the spot factor.

spot price of dollar, the 63-day forward rate (PRE), dollar implied rate (cupom cambial) and volatility. We denote them as

$$\xi_0 = (S, r_\$, r_{DOL}, \sigma_{DOL})$$

suppose that the risk scenario R corresponds to a shock of 3% on the exchange rate, a rise of 5% in interest rate, a drop of 2% in the cupom cambial and a rise of 50% in volatility on $t = 4$. In this case, we have

$$\xi_4 = (S(1.03), r_\$ + 0.05, r_{DOL} + 0.02, \sigma_{DOL}(1.50)).$$

The vector ξ_4 can be entered into the Black-Scholes formula to obtain the MTM value of the option on $t = 4$ according to this scenario.

2.3 Liquidation strategies

In this section, we define mathematically liquidation strategies and market liquidity constraints. These will be useful to formulate the search for a liquidation strategy as an optimization problem.

Consider a portfolio with positions in N products cleared by the CCP, and define q_{it} as the fraction of the position in instrument i which is liquidated at the close of day t. Then

$$n_{it} = 1 - \sum_{s=1}^{t-1} q_{is};$$

represents the fraction of the initial position in instrument i which is open at the start of business on day t.

To each position, we will associate a first date to initiate liquidation and a final liquidation date. These dates depend on the contract settlement date, in the case of listed contracts, and on the maximum number of contracts which can be traded in a day in an orderly market for each listed instrument (the market liquidity constraint). Contract expirations are also taken into account in the case of derivatives. OTC products may have further restrictions, such as the existence of dates in which the contract will be liquidated in an auction. We denote by T_{max} the largest of all such dates.In general, but not always, T_{max} will correspond to the auction date of one or more OTC instruments. We are led to restrictions on the variables q_{it}, namely

$$0 \leq \underline{q}_{it} \leq q_{it} \leq \overline{q}_{it} \tag{4}$$

$$\sum_{t=1}^{T_{max}} q_{it} = 1. \tag{5}$$

Equation (4) represents the *a priori* market liquidity constraints on the different contracts/instruments in the portfolio; equation (5) represents the fact that all contracts are liquidated at the close of business on the maximum date T_{max}. Notice that fraction on position i which has not been liquidated at the start of day t can also be written as

$$n_{it} = \sum_{s=t}^{T_{max}} q_{is}, \tag{6}$$

and that we have $n_{i1} = 1, n_{i\,T_{max}} = q_{i\,T_{max}}$.

2.4 P&L associated with a liquidation strategy

The unrealized profit/loss (PNL) between consecutive dates $t-1$ and t (in monetary units at date t) associated with instrument i is

$$\psi_i'(t, R) = Q_i \left[MTM_i(t, \xi_t) - e^{r\Delta t} MTM_i(t-1, \xi_{t-1}) \right], \tag{7}$$

where $\Delta t = 1/252$. This represents the change in the value of the position from one settlement date to another. We include the 1-day cost-of-carry through the discount factor $exp(r\Delta t) = expr(\xi_t(R))$, where r is the corresponding forward rate. We use the notation ψ_i' with a prime as a mnemonic that this is a difference of the MTM between consecutive dates. Another useful quantity is the present value of the difference between the value of the security at date t (or T+t) in some risk scenario and its value on the calculation date (t=0); this is given by

$$\psi_i(t,R) = Q_i \left[e^{-rt\Delta t} MTM_i(t,\xi_t) - MTM_i(0,\xi_0) \right]. \tag{8}$$

Suppose that we liquidate the position on date t. Then, $\psi_i(t,R)$ can be viewed as the *realized P&L* from liquidating the instrument i on date t if risk scenario R occurs. Clearly, we have

$$\psi_i(t,R) = \sum_{s=1}^{t} e^{-sr\Delta t} \psi_i'(s,R) \tag{9}$$

and

$$e^{-tr\Delta t} \psi_i'(t,R) = \psi_i(t,R) - \psi_i(t-1,R), \tag{10}$$

which, if we assume that the position is closed on date t, reflect the fact that the realized P&L at time t is (as it should be) the sum of daily marking-to-market changes between the calculation and the liquidation dates.

Let us apply these concepts at the portfolio level. Suppose that we fix a liquidation strategy $q = (q_{it})_{t=1}^{T_{max}}$. The MTM value of the portfolio of risky assets which remains at the beginning of date t is

$$V_t = \sum_{i=1}^{N} n_{it} Q_i MTM_i(t, \xi_t(R)), \tag{11}$$

and the cash account is the initial cash plus the P&L realized from liquidating positions on dates 1 to $t-1$.

For a given risk scenario R, the realized P&L from closing positions on date t is

$$\sum_{i=1}^{N} q_{it}\psi_i(t,R). \tag{12}$$

This amount is expressed in monetary units at time $t = 0$. Summing all the realized profits/losses due to liquidations on dates up and including t, we obtain the *realized P&L accumulated up to date* t (including trades done on date t) which can be written as

$$L_r(t,R,q) = \sum_{s=1}^{t}\sum_{i=1}^{N} q_{is}\psi_i(s,R). \tag{13}$$

At the beginning of date t (before trading) the cash account is equal to the original amount of cash plus $L_r(t-1,R,q)$.

In addition, the CCP should also consider *unrealized P&L* at time t, corresponding to the portion of the portfolio which has not been yet closed. The unrealized P&L at the start date t is given by

$$L_{nr}(t,R,q) = V_t - V_0 = \sum_{i=1}^{N} n_{it}\psi_i(t,R). \tag{14}$$

Therefore, the *total P&L up to date* t (after trading) is given by

$$L(t,R,q) = L_r(t-1,R,q)+L_{nr}(t,R,q) = L_r(t,R,q)+\sum_{i=1}^{N} n_{i(t+1)}\psi_i(t,R). \tag{15}$$

The potential shortfall for the CCP on date t is therefore

$$min\left[L(t,R,q)+V_0,0\right],$$

where we included the initial portfolio value, V_0.

3 Formulation of the CORE objective function

3.1 Worst-case loss estimates associated to a liquidation strategy

In order to safeguard the CCP during the liquidation process, we need to anticipate potential loss requirements at different dates, based on the choice of liquidation schedule q_{it}. This shall be done by subjecting portfolios held at different times to extreme risk scenarios on each date of the liquidation period.

A moment of reflection shows that, from a risk-management perspective, we should assume that the risk-scenarios affecting the portfolio on different dates are independent. This might seem excessively prudent; however, assuming independence avoids having to make subjective forecasts which might lead to under-estimating risk from the CCP perspective.

For spot risk-factors, we assume accordingly that any value for the risk factor within the band $[x_{min}(t), x_{max}(t)]$ is attainable on date t, and that the values associated to different dates are independent. For term-structure (curve) factors, any scenario among the possible deformations is possible on any valuation date of the liquidation period, and, as before, the deformations for different dates are independent.

Assume that we decided on a liquidation strategy $q = (q_{it})$. We apply extreme market scenarios for risk factors for each date to calculate potential losses and funding shortfalls.

The worst-case P&L on date t, evaluated over all risk scenarios, is the sum of the realized potential losses prior to date t and the unrealized potential loss on date t:

$$\min_R L(t,R,q) = \sum_{s=1}^{t-1} \min_{R_s} \left(\sum_{i=1}^{N} q_{is} \psi_i(s,R_s) \right) + \min_{R_t} \left(\sum_{i=1}^{N} n_{it} \psi_i(t,R_t) \right). \quad (16)$$

The subscript R_s in the risk scenario is used to emphasize that $\psi_i(s, R_s)$ depends only on the state of the market at time s, *i.e.*, $R_s = \xi_s(R)$, in the notation of Section 2.

We shall make a mild assumption concerning the risk-scenarios:

Assumption 1: *For any position in risky market instruments, there exists a risk-scenario for which unwinding the position gives rise to a loss.*[14]

Under this assumption, each of the summands in the last equation is non-positive.

In particular, the worst-case scenario for funding shortfalls on date t is

$$L_t(q) = \sum_{s=1}^{t-1} \left(\sum_{i=1}^{N} q_{is} \psi_i(s,R_s^*) \right) + \left(\sum_{i=1}^{N} n_{it} \psi_i(t,R_t^{**}) \right). \quad (17)$$

where

$$\begin{aligned} R_t^* &= \arg\min_{R_t} \left(\sum_{i=1}^{N} q_{it} \psi_i(t,R_t) \right) \\ R_t^{**} &= \arg\min_{R_t} \left(\sum_{i=1}^{N} n_{it} \psi_i(t,R_t) \right), \quad t = 1,2,3,...,T_{max}. \end{aligned} \quad (18)$$

Given any liquidation strategy q we can compute, in this way, a sequence of potential losses under stress scenarios for on different dates. We note, in particular, that the sequence of worst-case scenarios for realized losses is $R_t^*, t = 1, 2, 3, ...$; it is determined by "solving" T_{max} independent optimization problems. The

[14] By a "loss" we mean a non-positive P&L. This assumption will be satisfied if the set of risk-factors and risk-scenarios is sufficiently rich. Otherwise, we would have spurious arbitrage opportunities in the model.

same observation holds for the sequence for worst-case scenarios for unrealized losses $R_t^{**}, t = 1, 2, ..., T_{max}$.

3.2 Monotonicity of $L_t(q)$

Given the structure of risk-scenarios used in calculation of worst-case losses, we claim that $L_t(q) = \min_R L(t, R, q)$ is monotone decreasing. To see this, notice that

$$\begin{aligned}\sum_{i=1}^{N} n_{it}\psi_i(t, R_t^{**}) &= \sum_{i=1}^{N} q_{it}\psi_i(t, R_t^{**}) + \sum_{i=1}^{N} n_{i(t+1)}\psi_i(t, R_t^{**}) \\ &\geq \sum_{i=1}^{N} q_{it}\psi_i(t, R_t^{*}) + \sum_{i=1}^{N} n_{i(t+1)}\psi_i(t, R_t^{**}) \\ &\geq \sum_{i=1}^{N} q_{it}\psi_i(t, R_t^{*}) + \sum_{i=1}^{N} n_{i(t+1)}\psi_i(t+1, R_{t+1}^{**}). \qquad (19)\end{aligned}$$

The first inequality is justified by the fact that, by definition, R_t^* minimizes of the first term in the right-hand side (see equation (17)). The second inequality follows from the fact that the assumption of "expanding" risk scenarios implies that there is more risk for liquidating or marking-to-market on date $t + 1$ than on date t.

This monotonicity property has some interesting consequences. Most notably:

$$\begin{aligned}\min_{1\leq t\leq T_{max}} \min_R L(t, R, q) &= \min_R L(T_{max}, R, q) \\ &= \min_R L_r(T_{max}, R, q) \\ &= \sum_{t=1}^{T_{max}} \sum_{i=1}^{N} q_{it}\psi_i(t, R_t^{*}). \qquad (20)\end{aligned}$$

We have (due to the assumption on "expanding" risk scenarios)

The worst transient loss is equal to the terminal realized loss.

3.3 The CORE objective function

In this section, we define the mathematical optimization problem that we shall use to determine an optimal liquidation strategy. Recall that the worst-case loss at date t in scenario R for a liquidation schedule q is

$$L_t(q) = \sum_{s=1}^{t-1} \sum_{i=1}^{N} q_{is} \psi_i(s, R_s^*) + \sum_{i=1}^{N} n_{it} \psi_i(t, R_t^{**}). \tag{21}$$

The worst-case loss to the CCP on date t is estimated at $min[L_t^* + C_0 + V_0, 0]$, where we included the initial value of the portfolio. This is the amount that would have to be financed by the CCP on date t under the worst case realized/unrealized scenario. Therefore, the sum

$$\sum_{t=1}^{T_{max}} min[L_t(q) + V_0, 0] \tag{22}$$

represents the total estimated amount of funding needed during the liquidation period. We make the following assumptions.

Assumption 2: *The unwinding strategy should depend on the sequence of worst losses but not on the initial value of the portfolio.*

Assumption 3: *All other things being equal, a liquidation strategy q which requires less funding is more desirable than a strategy which requires more funding.*

Based on Assumption 2, we normalize the cash in the account so that:

$$V_0 = 0. \tag{23}$$

In this case, the sum of the daily funds needed to keep the account positive would be

$$\sum_{t=1}^{T_{max}} min[L_t(q), 0] = \sum_{t=1}^{T_{max}} \min_R L(t, R, q).$$

This suggests the use of the objective function:

$$\boxed{U(q) = \sum_{t=1}^{T_{max}} \min_R L(t, R, q).} \tag{24}$$

We note that other possible choices for the CORE utility function consistent with Assumptions 2 and 3 could be the worst potential transient loss over the liquidation period, *i.e.*

$$\begin{aligned} U_1(q) &= \min_{1 \leq t \leq T_{max}} \min_R L(t, R, q) = \min_R L(T_{max}, R, q) \\ &= \sum_{t=1}^{T_{max}} \sum_{i=1}^{N} q_{it} \psi_i(t, R_t^*). \end{aligned} \tag{25}$$

We give preference to the sum of the potential transient losses as objective function because it takes into account intermediate mark-to-market P&L (see the Introduction and the next section for practical examples).[15]

3.4 Concavity of $U(q)$ and Linear Programming

This section gives some mathematical properties of the objective function and shows why it is convenient for practical applications: namely the fact that it can be optimized using large-scale linear programming algorithms.

[15]Both objective functions are mathematically comparable since a simple calculation (which we omit here) gives

$$U_1(q) = \sum_{t=1}^{T_{max}} \sum_{i=1}^{N} q_{it} \psi_i(t, R_t^*) \leq \frac{1}{T_{max}} U(q) < \sum_{t=1}^{T_{max}} \left(1 - \frac{t}{T_{max}}\right) \sum_{i=1}^{N} q_{it} \psi_i(t, R_t^*)$$

which means that the objective function $U(q)$ "works" like terminal loss but, effectively, gives less penalty to late closeouts than $U_1(q)$.

Since $L(t, R, q)$ is linear in q, $\min_R L(t, R, q), t = 1, 2, ..., T_{max}$ is a concave function of q. This implies that the sum, $U(q)$,is also concave in q. Hence, determining the optimal liquidation strategy consists in maximizing a convex function under the linear equality/inequality constraints (4),(5).This is a well-posed problem optimization problem, which admits a unique solution, if there are sufficiently many risk-scenarios.

A major computational advantage of the objective function $U(q)$ is that, due to its special structure, the optimization problem is equivalent to maximizing a linear function under linear inequality constraints. In fact, following the duality principle in linear programming [6] , we introduce the $2 \times T_{max}$ auxiliary variables $L_1, ..., L_{T_{max}}$, and $M_1, ..., M_{T_{max}}$ and define the additional inequality constraints:

$$L_t \leq \sum_{i=1}^{N} (n_{it} - n_{i(t+1)} \psi_i(t, R) \quad t = 1, ..., T_{max}, \quad R \in \mathbf{R}$$

$$M_t \leq \sum_{i=1}^{N} n_{it} \psi_i(t, R) \quad t = 1, ..., T_{max}, \quad R \in \mathbf{R}. \tag{26}$$

The program becomes to maximize the sum

$$\sum_{t=1}^{T} [(T - t) L_t + M_t],$$

by varying $\{n_{it}, L_t, M_t\}$ subject to the constraints (26) and (4), (5), (the latter applied to $n_i t$ via linear transformation). We note, for comparison purposes, that the maximization of $U_1(q)$ under constraints can also be reduced to a linear-programming problem which consists in maximizing the sum

$$\sum_{t=1}^{T} L_t,$$

by varying $\{n_{it}, L_t$ subject to the constraints (26) (for L_t only) and (4), (5). Such problems can be solved for realistic portfolios with a large number of instruments and risk-scenarios, using high-performance optimizers such as CPLEX or Gurobi. The differences between $U(q)$ and $U_1(q)$ the that the former takes into account unrealized losses whereas the latter does not.

4 Analysis of CORE on Sample Portfolios

We performed the optimization of seven sample portfolios suggested by the research team of BM&F Bovespa. The idea was to evaluate the behavior of the CORE strategy (utility function $U(q)$) as well as the terminal loss utility $U_1(q)$, in concrete cases. In all examples, we mixed listed instruments with "T+2" settlement and large liquidity with OTC instruments with liquidity in 15 days (estimated auction date).

4.1 Sample Portfolios

Portfolio 1

Symbol	Product	Strike	Expiration	Quantity	T+k	Daily Limit
DOL_1	f	-	63	2000	2	500
DOL_1(call)	c	1.62	252	-2000	15	2000
DOL_1(put)	p	1.62	252	2000	15	2000

Table 1: Theoretical Portfolio 1 consists in a long position in USD futures and a short "conversion" in dollar options. The portfolio can be interpreted as being short dollar forward in a quantity equivalent to 2000 futures expiring in 1 year, with liquidation in $t = 15$, and a long position in USD futures expiring in 63 days, with liquidation in T+2 and daily limit of 500 contracts.

Portfolio 2

Symbol	Product	Strike	Expiration	Quantity	T+k	Daily Limit
DOL_1	f	-	63	2000	2	500
DOL_1(call)	c	1.62	63	2000	2	500
DOL_1(put)	p	1.62	63	-2000	2	500
DOL_1(call)	c	1.62	252	-2000	15	2000
DOL_1(put)	p	1.62	252	2000	15	2000

Table 2: Theoretical Portfolio 2 consists in (1) a long position in 2000 USD futures with expiration in 63 days, daily liquidity limit of 500 contracts, (2) a long position in a 50-delta conversion with expiration in 63 days, daily liquidity of 500 contracts, first settlement in T+2, (3) a short position in a 50-delta conversion with expiration in 252 days, equivalent to 2000 contracts, to be traded in t=15 for the full amount.

Portfolio 3

Symbol	Product	Strike	Expiration	Quantity	T+k	Daily Limit
DOL_1	f	-	63	2000	2	500
DOL_1(call)	c	1.62	63	2000	2	500
DOL_1(put)	p	1.62	63	-2000	2	500
DOL_1(call)	c	1.62	250	-2000	2	500
DOL_1(put)	p	1.62	250	2000	2	500
DOL_1(call)	c	1.62	252	-2000	15	2000
DOL_1(put)	p	1.62	252	2000	15	2000

Table 3: Theoretical Portfolio 3 consists in (1) a long position in 2000 USD futures with expiration in 63 days, daily liquidity limit of 500 contracts, (2) a long position in a 50-delta conversion with expiration in 63 days, daily liquidity of 500 contracts, first settlement on t=2, (3) a long position in 2000 conversions expiring in 250 days, with daily liquidity of 500 contracts (4) a short position in a 50-delta conversion with expiration in 252 days, daily liquidity of 2000 contracts to be traded on t=15 for the full amount.

Portfolio 4

Symbol	Product	Strike	Expiration	Quantity	T+k	Daily Limit
DOL_1	f	-	63	4000	2	500
DOL_1(call)	c	1.62	252	-2000	15	2000
DOL_1(put)	p	1.62	252	2000	15	2000

Table 4: Theoretical Portfolio 4 consists in (1) a long position in 4000 USD futures with expiration in 63 days, daily liquidity limit of 500 contracts, (2) a short position in a 50-delta conversion with expiration in 252 days, daily liquidity of 2000 contracts to be traded on t=15 for the full amount.

Portfolio 5

Symbol	Product	Strike	Expiration	Quantity	T+k	Daily Limit
DOL_1	f	-	63	2000	2	500
DOL_1(call)	c	1.62	252	-2000	15	2000
DOL_1(put)	p	1.62	252	-2000	15	2000

Table 5: Theoretical Portfolio 5 consists in (1) a long position in 2000 USD futures with expiration in 63 days, daily liquidity limit of 500 contracts, (2) a short position of 2000 in a 50-delta straddle with expiration in 252 days to be traded on t=15 for the full amount.

Portfolio 6

Symbol	Product	Strike	Expiration	Quantity	T+k	Daily Limit
DOL_1	f	-	63	2000	2	500
DOL_1(call)	c	1.62	63	2000	2	500
DOL_1(put)	p	1.62	63	2000	2	500
DOL_1(call)	c	1.62	252	-2000	15	2000
DOL_1(put)	p	1.62	252	-2000	15	2000

Table 6: Theoretical Portfolio 6 consists in (1) a long position in 2000 USD futures with expiration in 63 days, daily liquidity limit of 500 contracts, (2) a long position of 2000 in a 50-delta straddle with expiration in 63 days, daily limit of 500 contracts (3) a short position in 2000 50-delta straddles expiring in 252 days which must be closed-out in 15 days.

Portfolio 7

Symbol	Product	Strike	Expiration	Quantity	T+k	Daily Limit
DOL_1	f	-	63	2000	2	500
DOL_1(call)	c	1.62	63	2000	2	500
DOL_1(put)	p	1.62	63	2000	2	500
DOL_1(call)	c	1.62	220	2000	2	500
DOL_1(put)	p	1.62	220	2000	2	500
DOL_1(call)	c	1.62	252	-2000	15	2000
DOL_1(put)	p	1.62	252	-2000	15	2000

Table 7: Theoretical Portfolio 7 consists in (1) a long position in 2000 USD futures with expiration in 63 days, daily liquidity limit of 500 contracts, (2) a long position of 2000 50-delta straddle with expiration in 63 days, daily limit of 500 contracts, (2) a long position of 2000 50-delta straddle with expiration in 220 days, daily limit of 500 contracts, (3) a short position in 2000 50-delta straddles expiring in 252 days which must be closed-out in 15 days.

We note that Portfolios 1-4 are concerned with exposure in dollar and dollar futures and forwards (via option conversions) and differences in liquidities, whereas Portfolios 5-7 are concerned with exposure to volatility (via the use of straddles), as well as differences in liquidity.

An implicit assumption in this portfolio is that the contracts with liquidity in $t = 15$ must all be liquidated on date $t = 15$ through an auction. The remaining positions correspond to exchange-listed products. However, we assume that the latter have limited daily liquidity so cannot be used to hedge entirely the closeout on $t = 15$. Furthermore, some portfolios have redundancies or excess number of contracts. The goal of the experiment is to analyze how the choice of the objective function affects the close-out and, specifically, how the hedging and the redundancies are handled by CORE.

In all tests, we used four risk factors

1. DOL (spot)
2. Cupom cambial
3. Taxa PRE
4. USD/BRL implied volatility.

To simplify matters, we assumed flat term structure of volatility for the risk factors and had three shocks ('high', 'low', 'unchanged') per factor. We also used bands which widen progressively in time, consistently with BM&F Bovespa's approach.

The close-out strategies corresponding to each portfolio are computed next and discussed. We considered three different strategies for close out. First, the "naive strategy" in which the each position is closed as soon as possible according to the liquidity constraints; second, the $U_1(q)$ strategy, which closes out the portfolio so as to minimize the worst-case loss (which is also the terminal worst case loss) and, third, the $U(q)$ strategy in which we minimize the sum of the worst-case losses.

4.2 Portfolio 1

	Naïve			U1			U	
fut(63)	call(252)	put(252)	fut(63)	call(252)	put(252)	fut(63)	call(252)	put(252)
0	0	0	0	0	0	0	0	0
500	0	0	500	0	0	58	0	0
500	0	0	500	0	0	0	0	0
500	0	0	500	0	0	0	0	0
500	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	442	0	0
0	0	0	(0)	0	0	500	0	0
0	0	0	(0)	0	0	500	0	0
0	(2000)	2000	500	(2000)	2000	500	(2000)	2000

Figure 1: The optimal close-out strategies for different objective functions for Portfolio 1. We express the liquidation strategies in contract units as opposed to the dimensionless matrix q_{it}. To convert into q_{it} the reader should divide the entries by the initial position, indicated in Table 1, under the Quantity column.

The main difference between U_1 and U pertains to the liquidation of the 1500 contracts that cannot be liquidated simultaneously with the synthetic forward (due to daily limits). The question is: should these contracts be closed as soon as possible or near the auction date? Figure 1 shows that the objective function $U_1(q)$ favors early liquidation, whereas $U(q)$ favors liquidating closer to the auction date (for an amount comparable to the notional exposure in the forward), because this gives better MTM results – and thus less capital requirements – during the liquidation period. The naive strategy closes all positions as soon as possible.

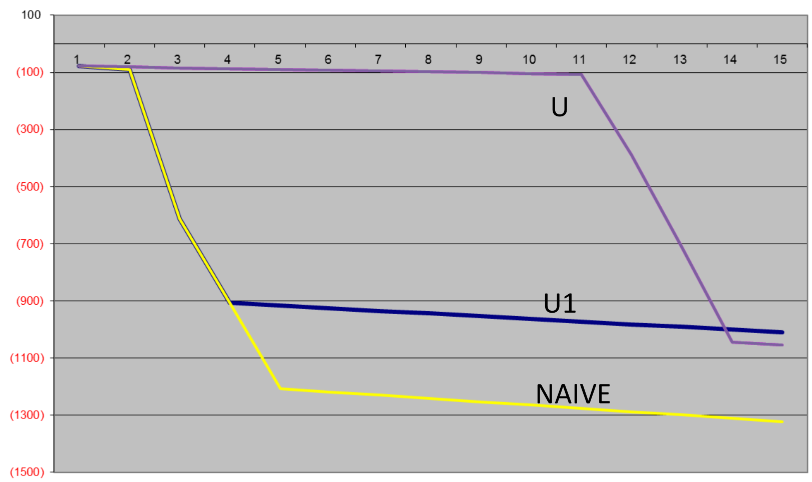

Figure 2: The transient losses (cumulative realized + unrealized) for the worst-case scenario for different strategies for Portfolio 1. Notice that the main difference between U_1 and U is that the latter has much smaller capital requirements than the optimal U_1-strategy along the liquidation periods, albeit with a slightly larger requirement at the end. Notice that the terminal requirements for U_1 and U are similar, and are much less severe than in the case of the naive strategy.

Figure 2 gives the transient loss curve according to the worst risk scenarios R_t^* for the strategy. The first remark from Figure 2 is that there is a 40% reduction in worst-case potential losses; CORE does indeed help in offsetting risk. The second remark is that $U(q)$ does slightly less well than $U_1(q)$ in terms of terminal loss due to the fact that, by definition, $U_1(q)$ gives the *best* result for terminal loss scenarios. Nevertheless, the transient losses under worst-case are much less in the case of $U(q)$ due the mark-to-model feature until the very end. This is why we prefer $U(q)$.

4.3 Portfolio 2

		Naïve				U1					U			
fut(63)	call(63)	put(63)	call(252)	put(252)	fut(63)	call(63)	put(63)	call(252)	put(252)	fut(63)	call(63)	put(63)	call(252)	put(252)
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
500	500	(500)	0	0	500	500	(500)	0	0	500	0	0	0	0
500	500	(500)	0	0	500	500	(500)	0	0	500	0	(0)	0	0
500	500	(500)	0	0	500	500	(500)	0	0	0	(0)	0	0	0
500	500	(500)	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	(0)	0	(0)	0	0
0	0	0	0	0	0	0	0	0	0	0	0	(0)	0	0
0	0	0	0	0	0	0	0	0	0	(0)	0	(0)	0	0
0	0	0	0	0	0	0	0	0	0	0	(0)	(0)	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	(0)	0	0	0	0
0	0	0	0	0	0	0	0	0	0	(0)	500	(500)	0	0
0	0	0	0	0	0	0	0	0	0	(0)	500	(500)	0	0
0	0	0	0	0	0	0	0	0	0	500	500	(500)	0	0
0	0	0	(2000)	2000	500	500	(500)	(2000)	2000	500	500	(500)	(2000)	2000

Figure 3: The optimal close-out strategies for different objective functions for Portfolio 2.

The optimal strategies for Portfolio 2 are displayed in Figure 3. We observe that CORE does much better than naive liquidation. Comparing among $U_1(q)$

and $U(q)$, we see that all strategies use 500 futures and 500 synthetic forwards to hedge on $t = 15$. This leaves us with an imbalance of 1000 futures. $U_1(q)$ ignores this and liquidates early. $U(q)$ schedules an additional long position in 1000 contracts (500 futures and 500 forwards) to be liquidated on T+14, to minimize MTM risk. After that, $U(q)$ further hedges the $t = 15$ risk by planning to close 500 futures and 500 synthetic forwards on T+14 In $U(q)$ the remaining balance of the long futures are unwound on T+13 and in T+2,T+3. Option spreads are not broken in the liquidation process.

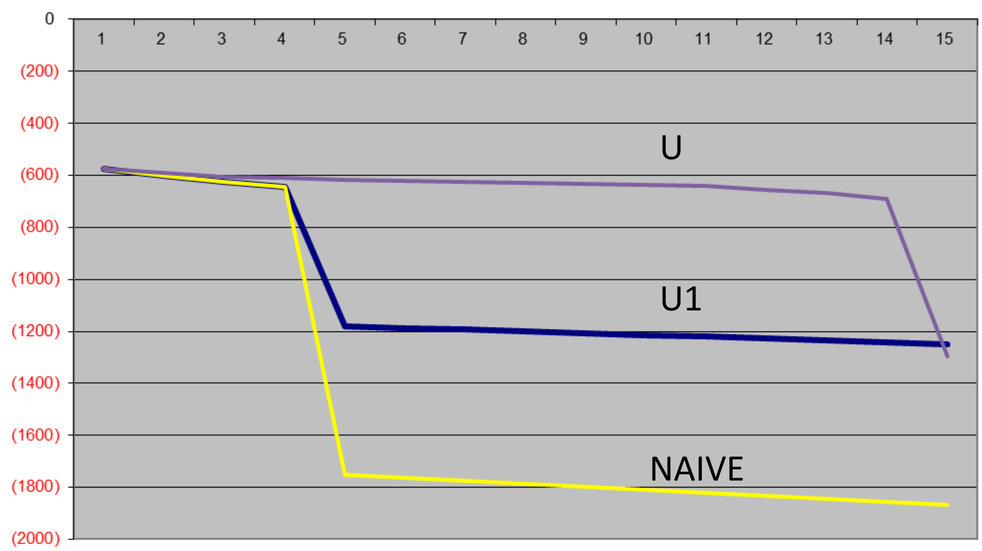

Figure 4: Worst-case transient loss function for Portfolio 2. Notice that CORE strategies cost 30% than the naive close-out strategy.

Looking at the transient loss curves and terminal values, one sees that (a)

CORE offers a significant improvement (more than 35% in this case) over naive unwinding of positions. Second, if we consider which strategy requires less interim capital. Here, we also see that $U(q)$ is better than strategy $U_1(q)$: its transient losses are much smaller for a longer period of time.

4.4 Portfolio 3

			Naïve							U1			
fut(63)	call(63)	put(63)	call(250)	put(250)	call(252)	put(252)	fut(63)	call(63)	put(63)	call(250)	put(250)	call(252)	put(252)
0	0	0	0	0	0	0	0	0	0	0	0	0	0
500	500	(500)	500	(500)	0	0	500	500	(500)	500	(500)	0	0
500	500	(500)	500	(500)	0	0	500	500	(500)	500	(500)	0	0
500	500	(500)	500	(500)	0	0	500	500	(500)	500	(500)	0	0
500	500	(500)	500	(500)	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	(2000)	2000	500	500	(500)	500	(500)	(2000)	2000

		U				
fut(63)	call(63)	put(63)	call(250)	put(250)	call(252)	put(252)
0	0	0	0	0	0	0
500	500	(500)	205	(213)	0	0
500	500	(500)	0	0	0	0
500	500	(500)	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	295	(287)	0	0
0	0	0	500	(500)	0	0
0	0	0	500	(500)	0	0
500	500	(500)	500	(500)	(2000)	2000

Figure 5: The optimal close-out strategies for different objective functions for Portfolio 3.

Figure 5 shows the close-out strategies for Portfolio 3. In this case the improvement between CORE/Naive is impressive. Also, $U_1(q)$ behaves “myopically” as usual. On the other hand, strategy $U(q)$ matches up to 1000 contracts on $t = 15$ (the maximum allowed due to liquidity.) In order to do this, it uses the long maturity synthetic. In other words, it is sensitive to the risk in the COUPON and PRE factors as well as to spot, and tries to match expirations. The remaining long expiration contracts are spread between early and late close-out. The short-term futures and synthetics, which offer no further offsets are liquidated early.

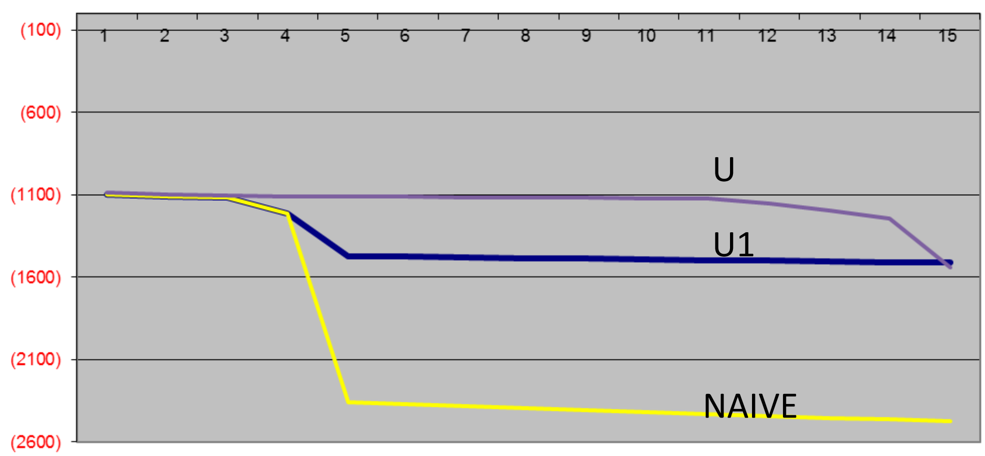

Figure 6: Worst–case transient loss function for Portfolio 3. Notice that CORE strategies cost ~50% than the naive close-out strategy
.

In Figure 6 we observe that the strategy $U(q)$ requires less interim funds than $U_1(q)$ and converges slowly to the terminal value.

4.5 Portfolio 4

	Naïve			U1			U	
fut(63)	call(252)	put(252	fut(63)	call(252)	put(252	fut(63)	call(252)	put(252
0	0	0	0	0	0	0	0	0
500	0	0	500	0	0	500	0	0
500	0	0	500	0	0	500	0	0
500	0	0	500	0	0	500	0	0
500	0	0	500	0	0	500	0	0
500	0	0	500	0	0	0	0	0
500	0	0	500	0	0	0	0	0
500	0	0	500	0	0	0	0	0
500	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	500	0	0
0	0	0	0	0	0	500	0	0
0	0	0	0	0	0	500	0	0
0	(2000)	2000	500	(2000)	2000	500	(2000)	2000

Figure 7: The optimal close-out strategies for different objective functions for Portfolio 4.

Figure 7 contains the results for Portfolio 4. This is similar to Portfolio 1, but with more directional exposure in the futures. As expected, $U(q)$ uses all futures necessary to eliminate MTM risk for as long as possible, additional inventory is liquidated early. Figure 8 has the corresponding transient loss curves.

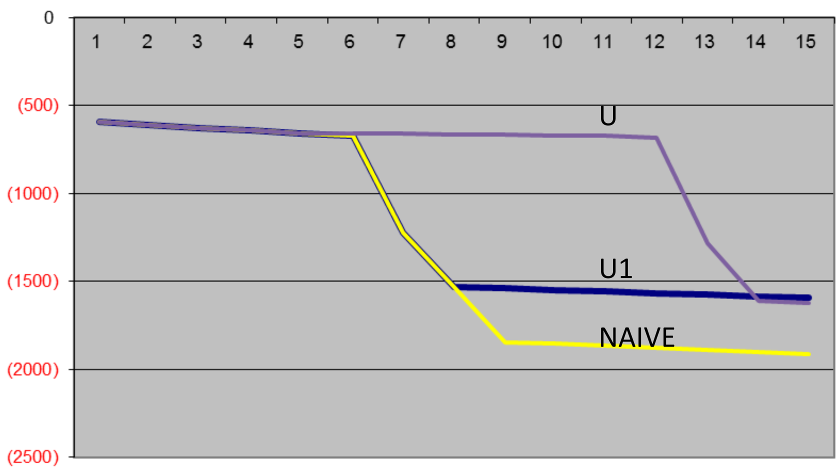

Figure 8: Worst–case transient loss functions for Portfolio 4.

	Naïve			U1			U	
fut(63)	call(252)	put(252)	fut(63)	call(252)	put(252)	fut(63)	call(252)	put(252)
0	0	0	0	0	0	0	0	0
500	0	0	500	0	0	500	0	0
500	0	0	500	0	0	500	0	0
500	0	0	500	0	0	45	0	0
500	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	453	0	0
0	(2000)	(2000)	500	(2000)	(2000)	500	(2000)	(2000)

Figure 9: The optimal close-out strategies for different objective functions for Portfolio 5.

4.6 Portfolio 5

Portfolio 5 is a short-volatility, long deltas portfolio. As shown in Figure 9, $U(q)$ liquidates the delta 50% at the beginning and 50% at the end. $U_1(q)$ liquidates all deltas immediately. Figure 10 has the corresponding transient loss curves.

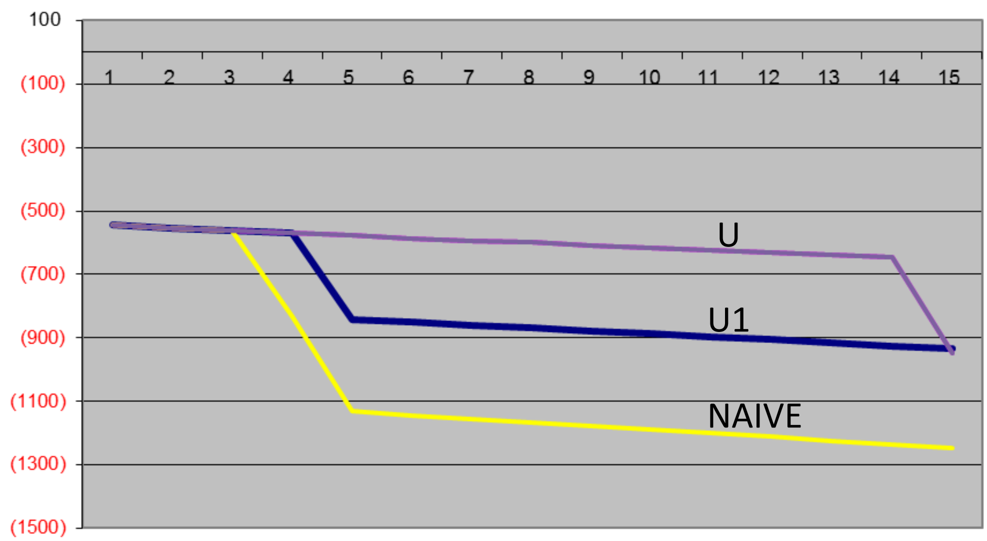

Figure 10: Worst cases transient loss functions for Portfolio 5.

4.7 Portfolio 6

		Naïve					U1		
fut(63)	call(63)	put(63)	call(252)	put(252)	fut(63)	call(63)	put(63)	call(252)	put(252)
0	0	0	0	0	0	0	0	0	0
500	500	500	0	0	479	0	500	0	0
500	500	500	0	0	479	0	500	0	0
500	500	500	0	0	390	0	500	0	0
500	500	500	0	0	152	0	196	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	500	104	0	0
0	0	0	0	0	0	500	101	0	0
0	0	0	0	0	0	500	98	0	0
0	0	0	(2000)	(2000)	500	500	0	(2000)	(2000)

		U		
fut(63)	call(63)	put(63)	call(252)	put(252)
0	0	0	0	0
500	0	443	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
46	0	57	0	0
319	500	500	0	0
323	500	500	0	0
312	500	500	0	0
500	500	0	(2000)	(2000)

Figure 11: The optimal close-out strategies for different objective functions for Portfolio 6.

Portfolio 6 has the first test for hedging volatility (listed versus OTC, say). The portfolio is short a 63-day straddle (listed) and an 252-day straddle (OTC). The liquidation schedules are given in Figure 11. Figure 12 contains the transient loss curves, which suggest that in this particular case, the loss function for $U(q)$ is nearly equal to the loss function for $U_1(q)$ – resulting in nearly-optimal close-out from the point of view of transient losses (Figures 11 and 12).

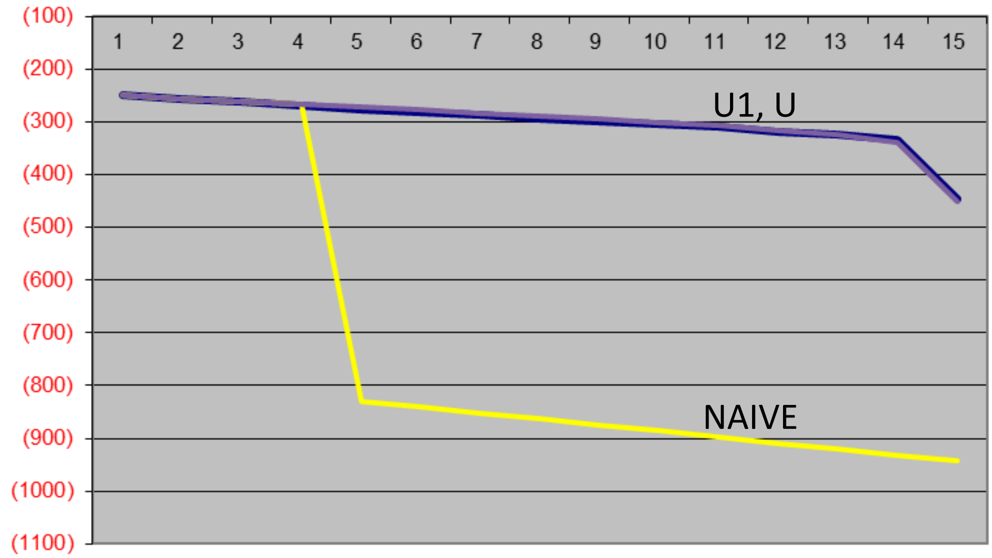

Figure 12: Worst cases transient loss functions for Portfolio 6.

4.8 Portfolio 7

			Naïve						U1				
fut(63)	call(63)	put(63)	call(220)	put(220)	call(252)	put(252)	fut(63)	call(63)	put(63)	call(220)	put(220)	call(252)	put(252)
0	0	0	0	0	0	0	0	0	0	0	0	0	0
500	500	500	500	500	0	0	500	0	427	0	36	0	0
500	500	500	500	500	0	0	253	0	285	21	0	0	0
500	500	500	500	500	0	0	0	0	0	0	0	0	0
500	500	500	500	500	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	23	0	32	5	0	0	0
0	0	0	0	0	0	0	360	0	500	74	0	0	0
0	0	0	0	0	0	0	362	0	500	74	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	2	500	0	326	464	0	0
0	0	0	0	0	0	0	0	500	65	500	500	0	0
0	0	0	0	0	0	0	0	500	55	500	500	0	0
0	0	0	0	0	(2000)	(2000)	500	500	136	500	500	(2000)	(2000)

		U				
fut(63)	call(63)	put(63)	call(220)	put(220)	call(252)	put(252)
0	0	0	0	0	0	0
500	0	499	0	9	0	0
479	0	500	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
357	0	500	74	0	0	0
151	0	210	31	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	500	20	395	491	0	0
0	500	65	500	500	0	0
12	500	70	500	500	0	0
500	500	136	500	500	(2000)	(2000)

Figure 13: Worst cases transient loss functions for Portfolio 7.

Portfolio 7 has 4,000 long straddles and 2,000 short straddles; it also has directional futures positions (2000 contracts). The optimal liquidation schedule consists in breaking up some of the short-dated (63-day) straddles and closing them out so as to minimize directional (USD/BRL) risk throughout the entire liquidation. For example the position consisting in 500 futures and 500 straddles, is closed by liquidating first 500 futures and 500 puts and keeping the 500 calls open. This strategy has premium risk, but is protected against large spot moves (see Figure 14). The 500 calls provide protection against a rise in the market

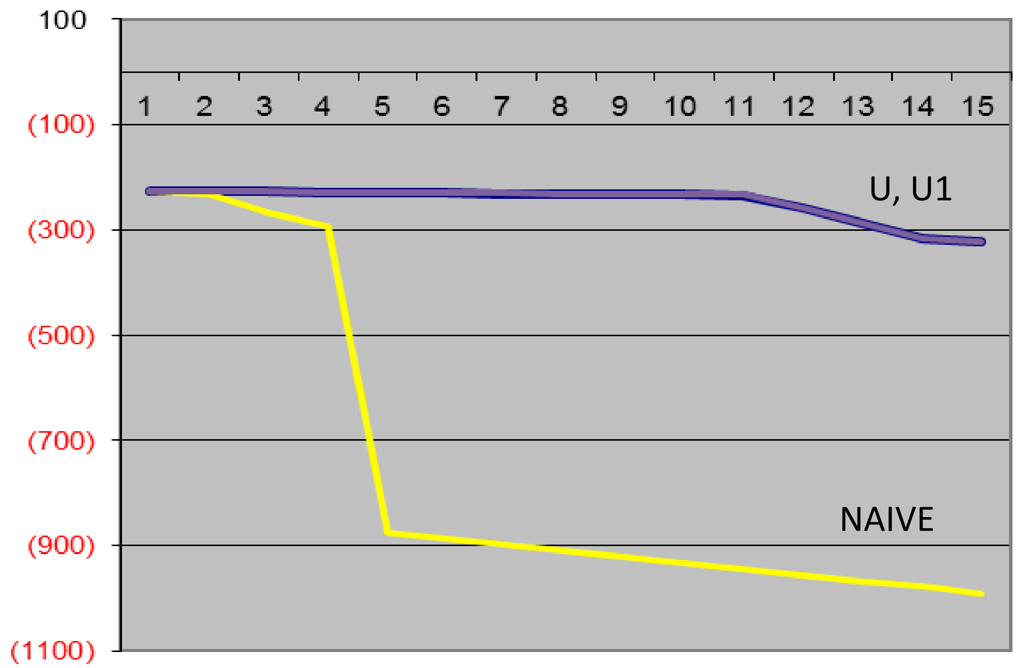

Figure 14: The optimal close-out strategies for different objective functions for Portfolio 7.

and/or a rise in volatility. The 220-day long volatility position is closed on $t = 15$ and T+14 to hedge the 2000 short straddles that must be liquidated on $t = 15$. During the close-out strategy, the portfolio is such that there is no unbounded risk in the case of a large USD move. This close-out strategy produces small transient losses, as shown in Figure 13.

4.9 Comparison between CORE ($U(q)$), $U_1(q)$ and naive liquidation requirements

We present the capital requirements associated with the worst terminal loss $\min_R L(T_{max}, R, q)$ in the three frameworks. The objective is to estimate the reduction in capital requirements that might be achieved by the use of CORE *vis a vis* naive liquidation. The results, which were obtained in the analysis of the different portfolios is presented in Table 8

Worst-case scenario losses

Portfolio	Naive	CORE ($U(q)$)	Improvement
1	1322	1053	21 %
2	1869	1298	31 %
3	2476	1541	38 %
4	1913	1621	16 %
5	1246	947	24 %
6	943	451	53 %
7	990	322	68 %

Table 8: Worst-case scenario losses for the different portfolios, assuming (i) naive liquidation and (ii) CORE. The results are expressed in the same monetary units as those used in the portfolios in the beginning of Section 4. The last column represents the reduction in terminal worst loss if we use the CORE schedule as opposed to the naive liquidation schedule.

5 Conclusions

We discussed a new proposal for a risk-management system for Central Counterparties, the Close-Out Risk Evaluation or CORE. This approach is based on specifying liquidity constraints explicitly for the various instruments in the portfolio and, based on risk-scenarios, search for an appropriate close-out, or liquidation, strategy for the portfolio. The requirements of this close-out strategy is that it takes into account the common risk-factors associated with the different positions. By seeking to liquidate simultaneously instruments which are subject to the same risk-factors, it mitigates market risk for the CPP and, ultimately, will require less margin. On the other hand, the strategy must take into account the liquidity constraints for the different instruments, including OTC instruments that will be liquidated via auctions or bilateral arrangements with solvent participants.

This required the proposal of an *objective function* which was simple to optimize and which took into account the CCP's preferences in terms of scheduling the close-out of a portfolio. The main contribution that was made is the proposal of an objective function and a simple but powerful algorithm, based on linear programming, which determines an "optimal" close-out strategy. We selected as objective function the *sum of the worst-case transient losses*

$$U(q) = \sum_{t=1}^{T} \min_{R} L(t, R, q). \tag{27}$$

where $L(t, R, q)$, defined in (**??**), represents the funding requirement generated by the strategy q at date t in risk scenario R.

The minimization of this objective function is simple to implement and has desirable properties in terms of balancing realized losses and MTM losses. The idea of using the sum of the worst-case transient losses, as opposed to just using the worst-case loss, is to minimize funding requirements over the liquidation period, by taking into account requirements associated with both realized losses and MTM losses across time. The fact that the objective function requires less funding for the strategy is useful in practice due to the fact that the liquidity constraints and auction dates are estimated parameters. For instance, using CORE strategies will result in optimal requirements in case OTC instruments can be liquidated before the (worst-case) auction date.

We showed that the proposed objective function (27) may be minimized using Linear Programming (LP); this enables well-understood high-performance numerical methods [7] to be used for computing optimal closeout strategies and makes the implementation feasible for large, complex portfolios.

We demonstrated the feasibility and adequacy of the approach by applying it to a wide range of sample portfolios and comparing the results with those obtained using the alternative “worst-loss” formulation as well as with a “naive” liquidation which does not take into account risk-offsets. The latter method is, to some extent, what is currently used by most clearinghouses. We analyzed 7 portfolios and found that the improvement in worst-case losses between CORE and naive liquidation are very significant, ranging from $21\% to 68\%$ in the examples that we analyzed. This suggests that CORE may lead to an efficient use of CCP capital without increasing risk with respect to standard risk-methodologies which are currently in place.

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