Risk and Portfolio Management
Spring 2010

Option portfolios with
several underlying assets

Option trades and portfolios: Many different styles

-- Carry trades using options (implied dividend vs. actual dividend, HTB)

-- Volatility surface trades (non-directional): trading different strikes on the same underlying asset

-- historical vol vs implied vol

-- Relative-value trades across names (non-directional)

  -- single-name option versus fair-value
  -- dispersion trading (index option versus components)

-- Directional volatility trades (long vol/ short vol, etc)

Delta-neutral option position

-- Open position (long or short) and simultaneously trade the stock so as to be delta-neutral.
-- Adjust the Delta of the option as the stock/option prices move

$$dC = \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S} dS + \frac{\partial C}{\partial \sigma} d\sigma + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} dS^2 + \ldots$$

$$P\,\&\,L \approx dC - \Delta dS + \Delta S r dt - \Delta S d dt - rC dt$$

$$= \left(\frac{\partial C}{\partial S} - \Delta \right) dS + \frac{\partial C}{\partial \sigma} d\sigma + \frac{S^2}{2} \frac{\partial^2 C}{\partial S^2} \left(\frac{dS^2}{S^2} - \sigma^2 dt \right)$$

$$- \left(\frac{\partial C}{\partial S} - \Delta \right) S (r - d) dt$$

$$+ \left(\frac{\partial C}{\partial t} + \frac{S^2 \sigma^2}{2} \frac{\partial^2 C}{\partial S^2} + (r - d) S \frac{\partial C}{\partial S} - rC \right) dt$$

$$\approx \frac{\partial C}{\partial \sigma} d\sigma + \frac{S^2}{2} \frac{\partial^2 C}{\partial S^2} \left(\frac{dS^2}{S^2} - \sigma^2 dt \right)$$

Book-keeping: profit/loss from a delta-hedged option position

$$\text{P/L} = \theta \cdot \left(n^2 - 1\right) + V \cdot d\sigma \qquad \left(n = \frac{1}{\sigma\sqrt{dt}}\frac{dI}{I}\right)$$

or

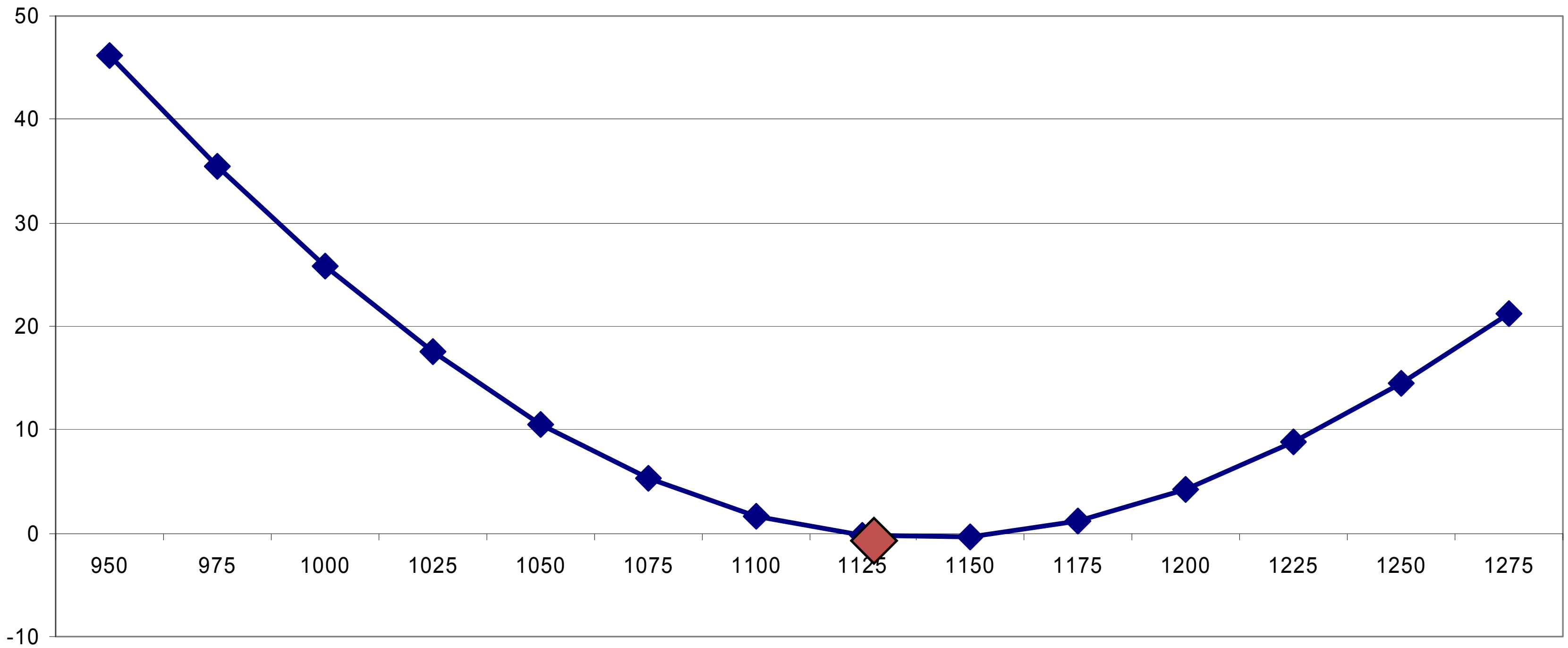
$$\text{P/L} = \frac{1}{2}\Gamma \cdot \left(\frac{(dI)^2}{I^2} - \sigma^2 dt\right) + V \cdot d\sigma$$

Gamma-Theta exposure

Vega exposure

1-day P/L for Long Call/Short Stock

(Constant volatility=16%)

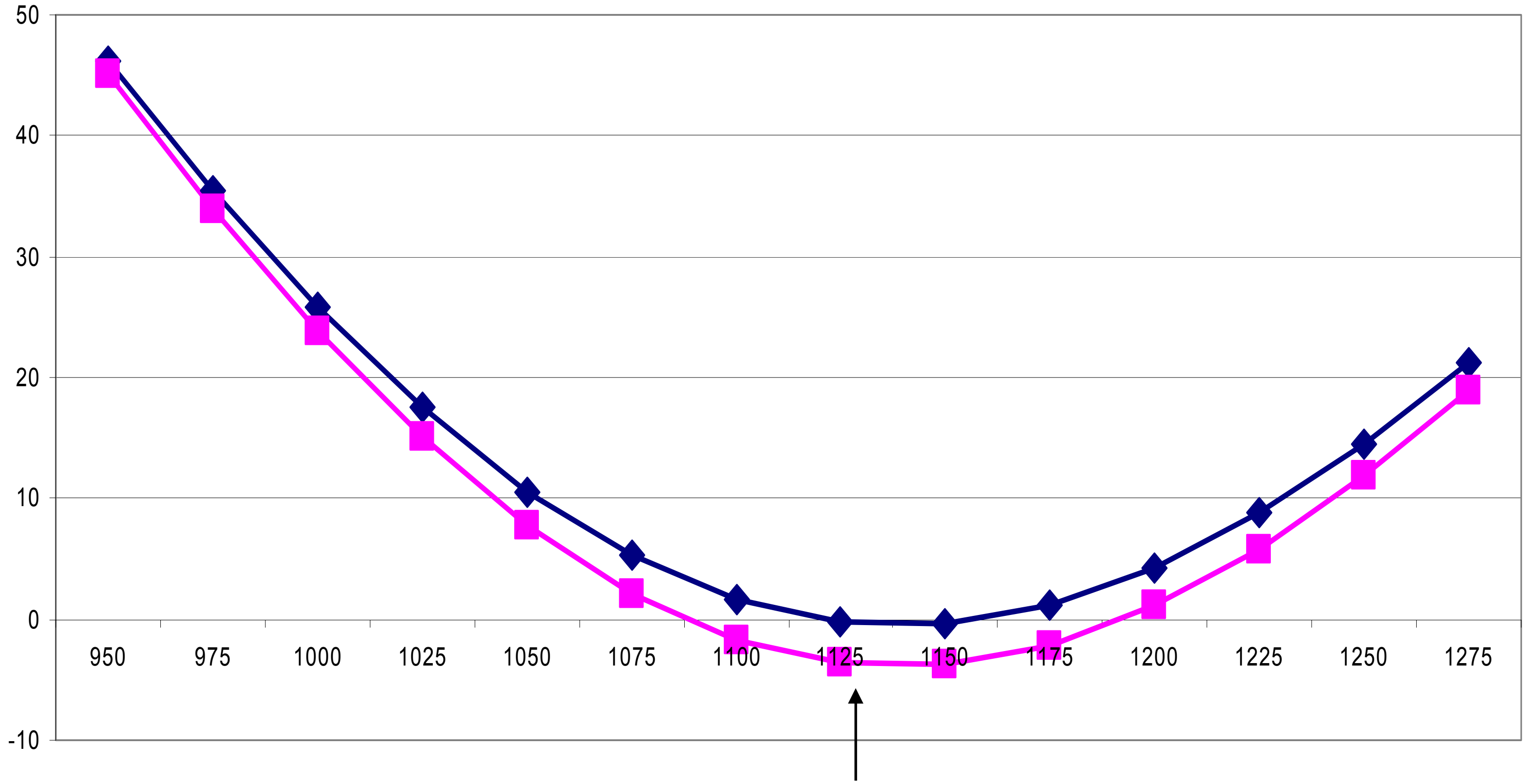


$$\text{P/L} \approx \theta \cdot \left(n^2 - 1\right)$$

$$\theta = \text{daily time-decay}, \quad n = \frac{\text{percent index change}}{\text{expected daily volatility}}$$

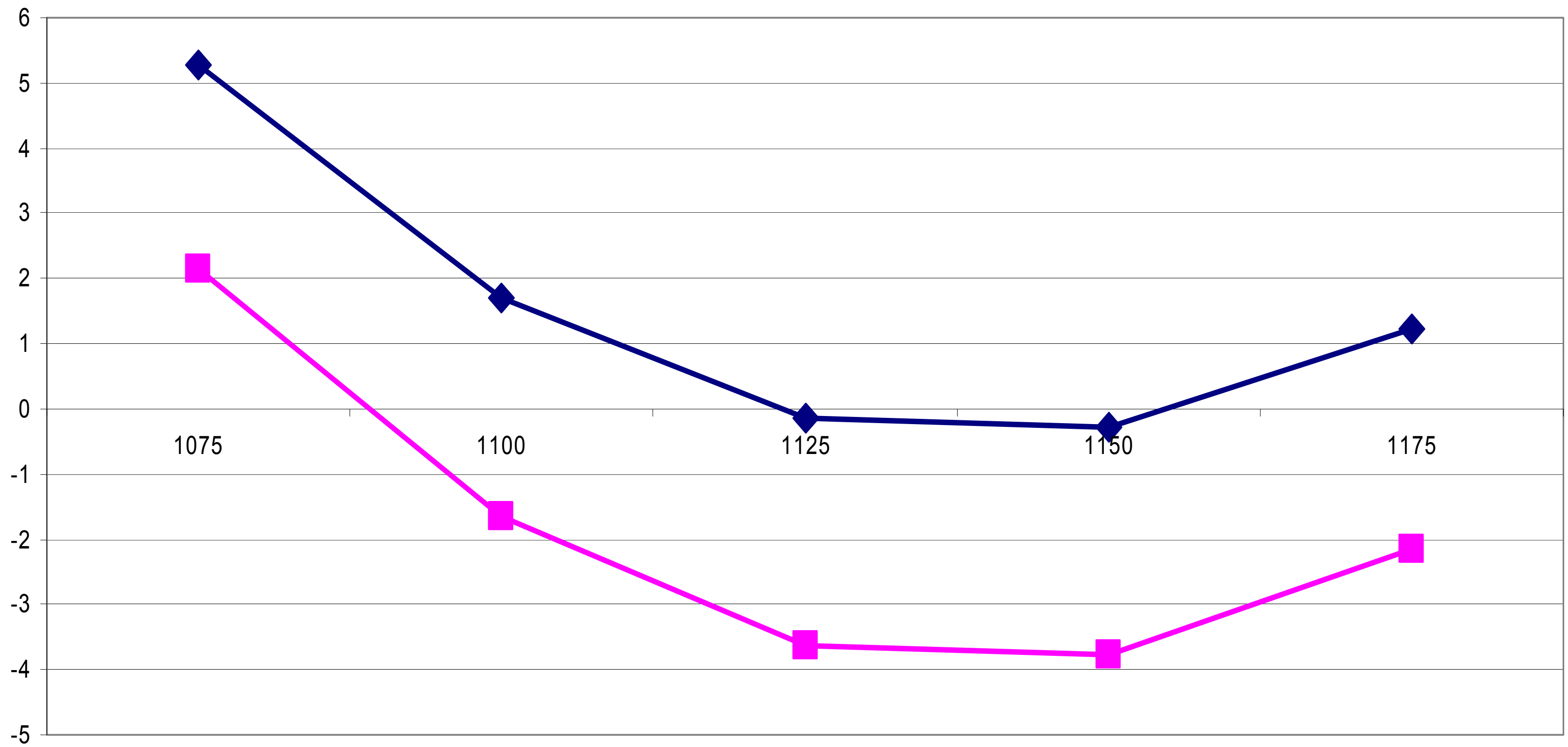
Assuming an implied volatility drop of 1%

Vol=15%



3.80 loss if stock does not move and volatility drops 1%

A closer look at the profit-loss due to a change in volatility



1% move in vol => 8% move in premium for a 6m ATM option

Measuring the Risk of a Portfolio (assuming delta neutrality)

Portfolio of options on N stocks

n_{ij} contracts of option with underlying

stock i, expiration T_j, volatility σ_{ij}

$$\Delta\Pi = \sum_{ij} n_{ij}\left(C\left(S_i + \Delta S_i, T_j, K_{ij}, \sigma_{ij} + \Delta\sigma_{ij}\right) - C\left(S_i, T_j, K_{ij}, \sigma_{ij}\right) - \frac{\partial C_{ij}}{\partial S_i}\Delta S_i \right)$$

$$= \sum_{ij} n_{ij}\left(C\left(S_i(1+R^{S_i}), T_j, K_{ij}, \sigma_{ij}\left(1+R^{\sigma_{ij}}\right)\right) - C\left(S_i, T_j, K_{ij}, \sigma_{ij}\right) - \frac{\partial C_{ij}}{\partial S_i} S_i R^{S_i} \right)$$

Need to define a <u>joint distribution</u> of stock returns and volatility returns to calculate statistics of PNL

Factor Models for Price/Vols

Consider only parallel vol shifts and use 30-day ATM volatilities

$$R^{S_i} = \sum_{k=1}^{m} \beta_{ik} F_k + \varepsilon_i$$

$$R^{\sigma_i} = \sum_{k=1}^{m} \gamma_{ik} F_k + \varsigma_i$$

Extract factors from PCA of augmented matrix

$$C_{ij} = \left\langle R^{S_i} R^{S_j} \right\rangle, \;\; D_{ij} = \left\langle R^{S_i} R^{\sigma_j} \right\rangle, \;\; E_{ij} = \left\langle R^{\sigma_i} R^{\sigma_j} \right\rangle$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{C} & \mathbf{D} \\ \mathbf{D'} & \mathbf{E} \end{pmatrix} \qquad \mathbf{M} \in R^{2N \times 2N}$$

Multivariate Analysis of Implied Vols

-- ATM constant maturity vols can be built using interpolation of variances

$$\sigma_{30d}^2 = \frac{30 - T_1}{T_2 - T_1}\sigma_{T_1}^2 + \frac{T_2 - 30}{T_2 - T_1}\sigma_{T_2}^2$$

-- WRDS has historical data on CM volatility surfaces parameterized by Deltas for standard maturities (*Option Metrics*)

-- Compute extreme values of standardized vol returns

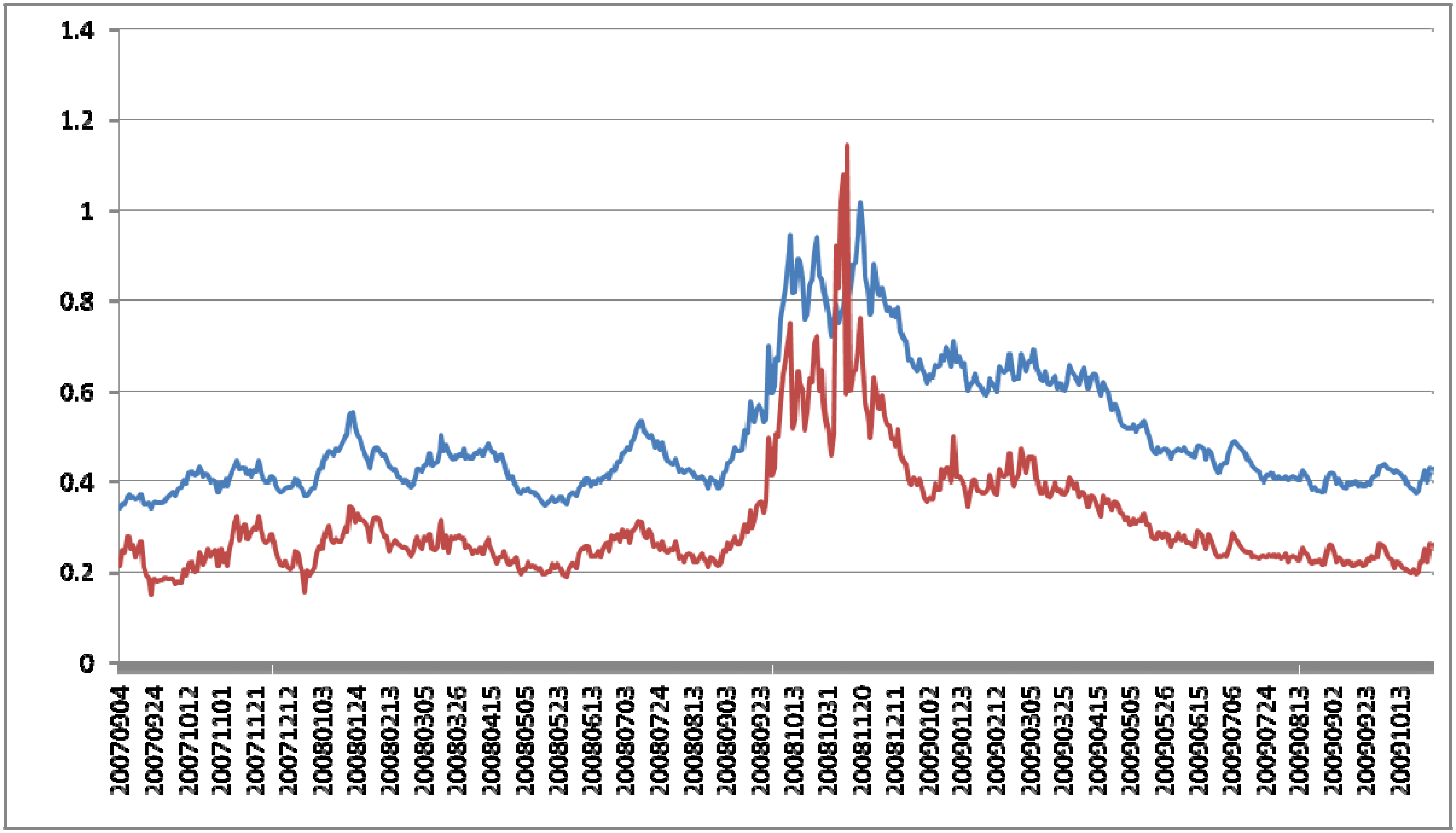
-- Perform <u>factor</u> analysis (PCA) to explore the dimensionality of the cross-section

-- Dataset: 98 constituents of Nasdaq 100, from 9/4/2008 to 10/30/2009

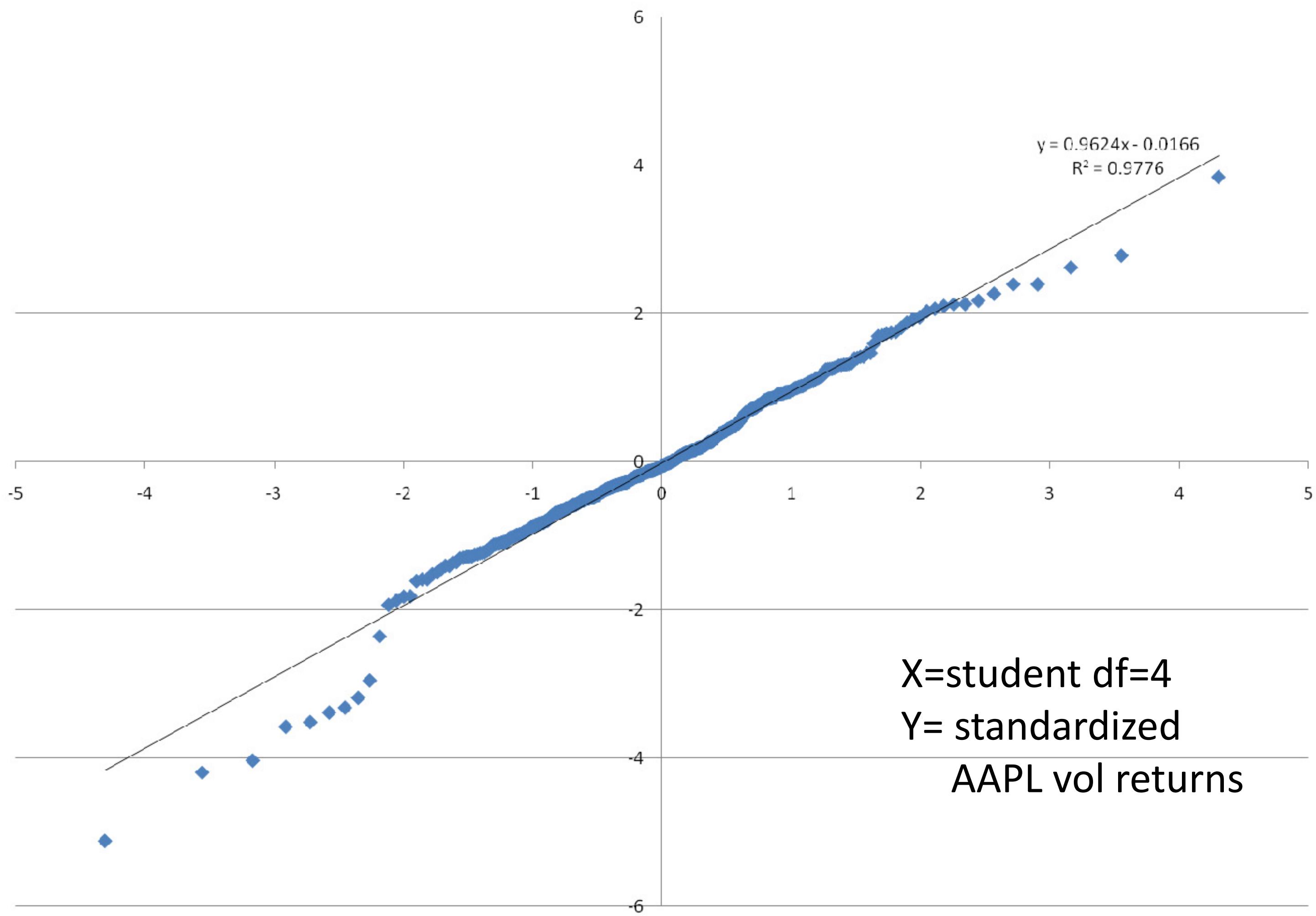
Excerpt of the data used for the calculations

DATES	AAPL	ADBE	ADSK	AKAM	ALTR	AMAT	AMGN	AMLN	AMZN	APOL
20070904	45.2%	30.9%	32.7%	42.9%	30.4%	27.9%	29.4%	44.9%	37.9%	38.8%
20070905	48.0%	29.5%	32.3%	44.7%	31.0%	29.1%	31.3%	44.8%	41.1%	39.2%
20070906	45.7%	29.6%	31.9%	46.6%	30.9%	28.7%	31.6%	45.6%	39.6%	39.5%
20070907	46.2%	32.2%	33.8%	46.7%	32.0%	33.1%	32.9%	47.1%	40.4%	40.3%
20070910	45.6%	33.6%	34.3%	45.0%	32.7%	33.2%	33.5%	47.7%	41.8%	43.0%
20070911	45.9%	32.5%	33.3%	42.8%	31.3%	32.1%	27.8%	47.6%	41.0%	41.9%
20070912	44.5%	32.7%	34.0%	42.5%	31.9%	33.4%	26.7%	46.5%	41.3%	42.8%
20070913	43.1%	34.6%	33.6%	41.8%	31.3%	32.7%	25.1%	49.5%	42.3%	43.0%
20070914	42.1%	34.0%	32.6%	43.0%	31.4%	32.9%	27.6%	46.6%	42.2%	42.7%
20070917	44.2%	36.0%	33.9%	45.8%	34.2%	32.3%	27.9%	49.7%	43.9%	45.1%
20070918	40.1%	26.8%	30.3%	44.3%	29.1%	31.3%	25.7%	49.8%	42.2%	44.4%
20070919	39.8%	26.1%	31.9%	44.3%	29.7%	29.7%	28.2%	48.4%	41.0%	42.5%
20070920	38.5%	27.5%	31.3%	43.2%	29.6%	30.4%	27.5%	47.8%	42.5%	43.4%

Average Implied Volatility vs. QQV
(Implied Vol of NDX-100)

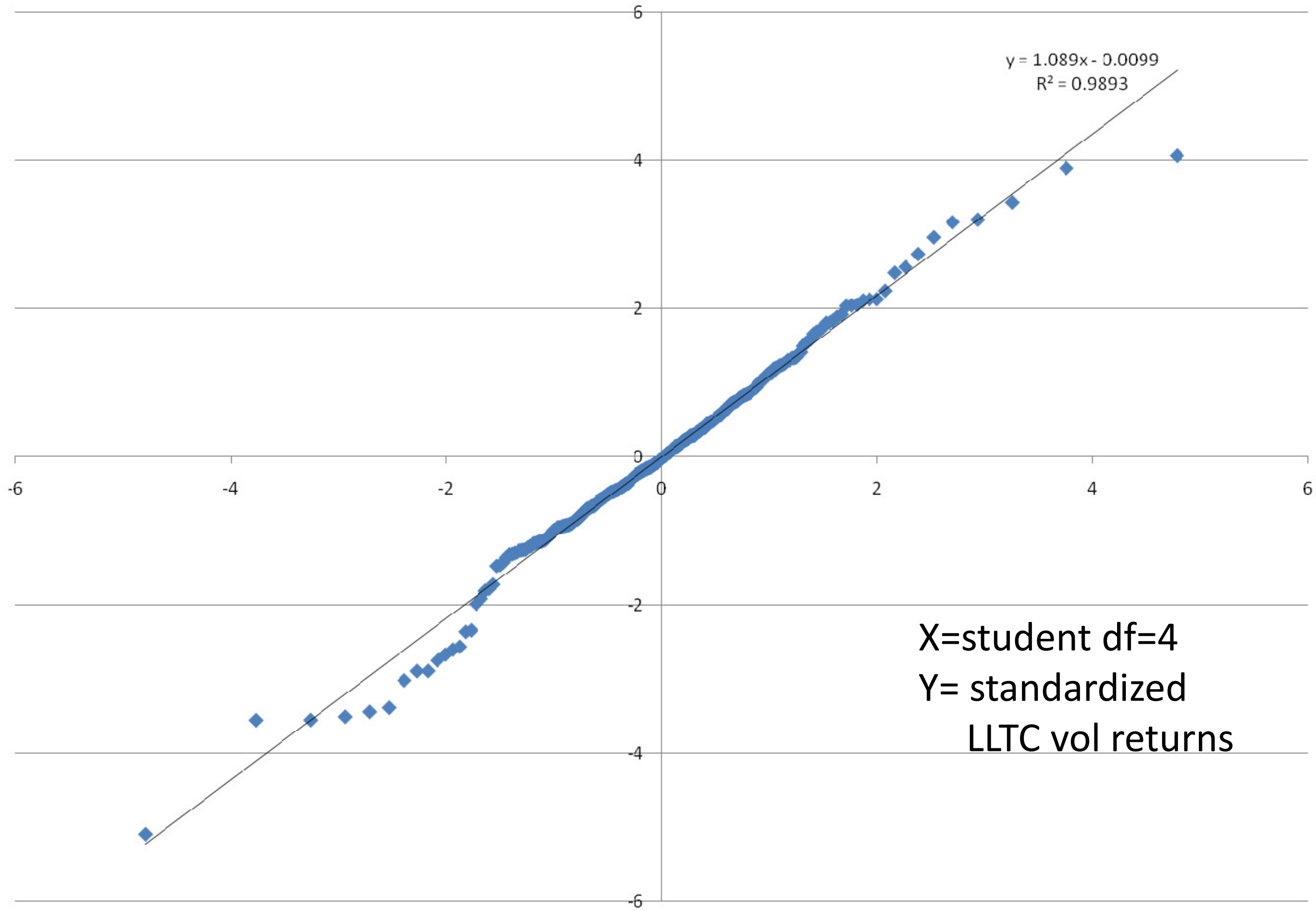


QQ-plot: AAPL 30D vol shocks

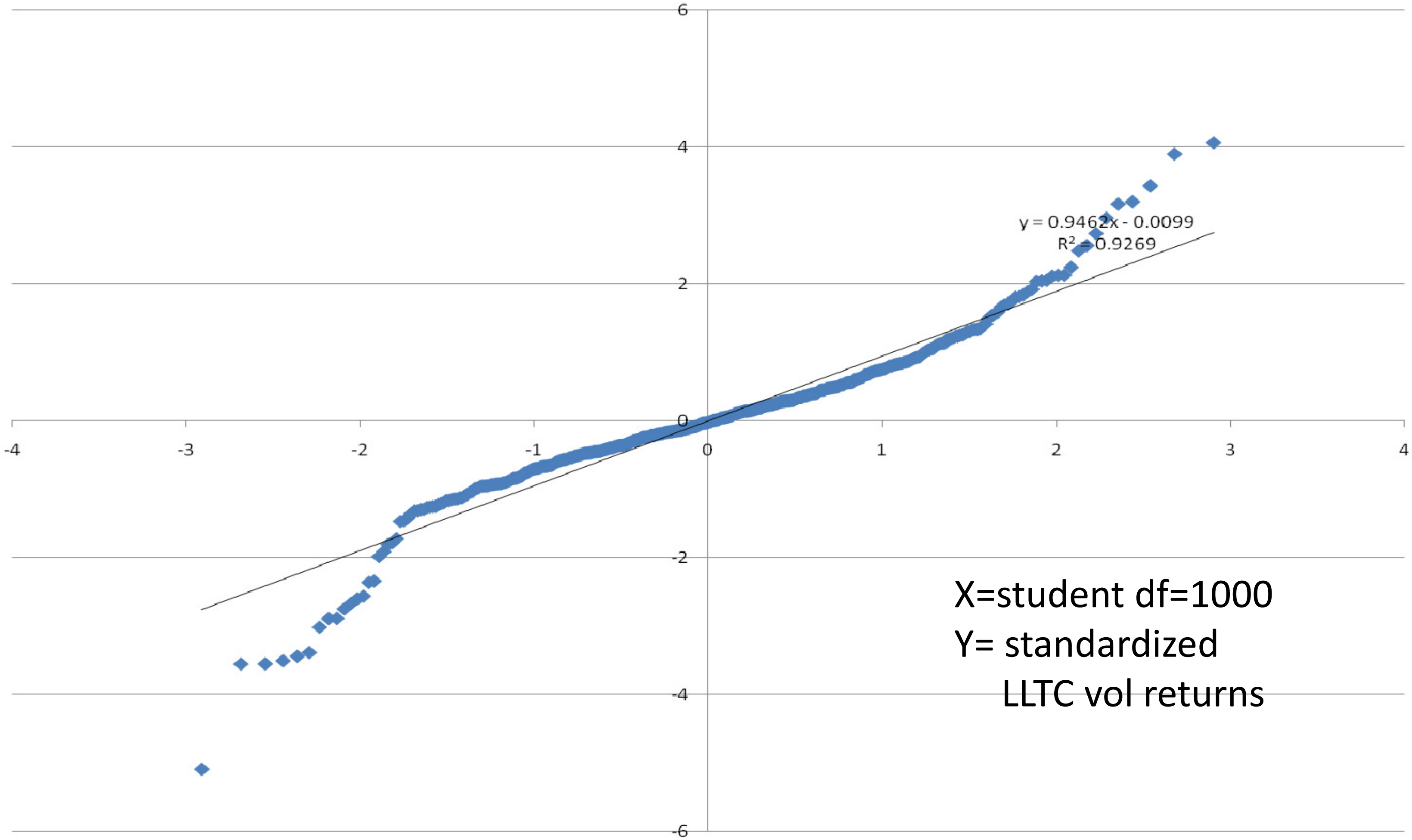


X=student df=4
Y= standardized
AAPL vol returns

QQ-plot: LLTC vol returns



LLTC vs Student with df=1000
(just to see that tails are indeed fat!)



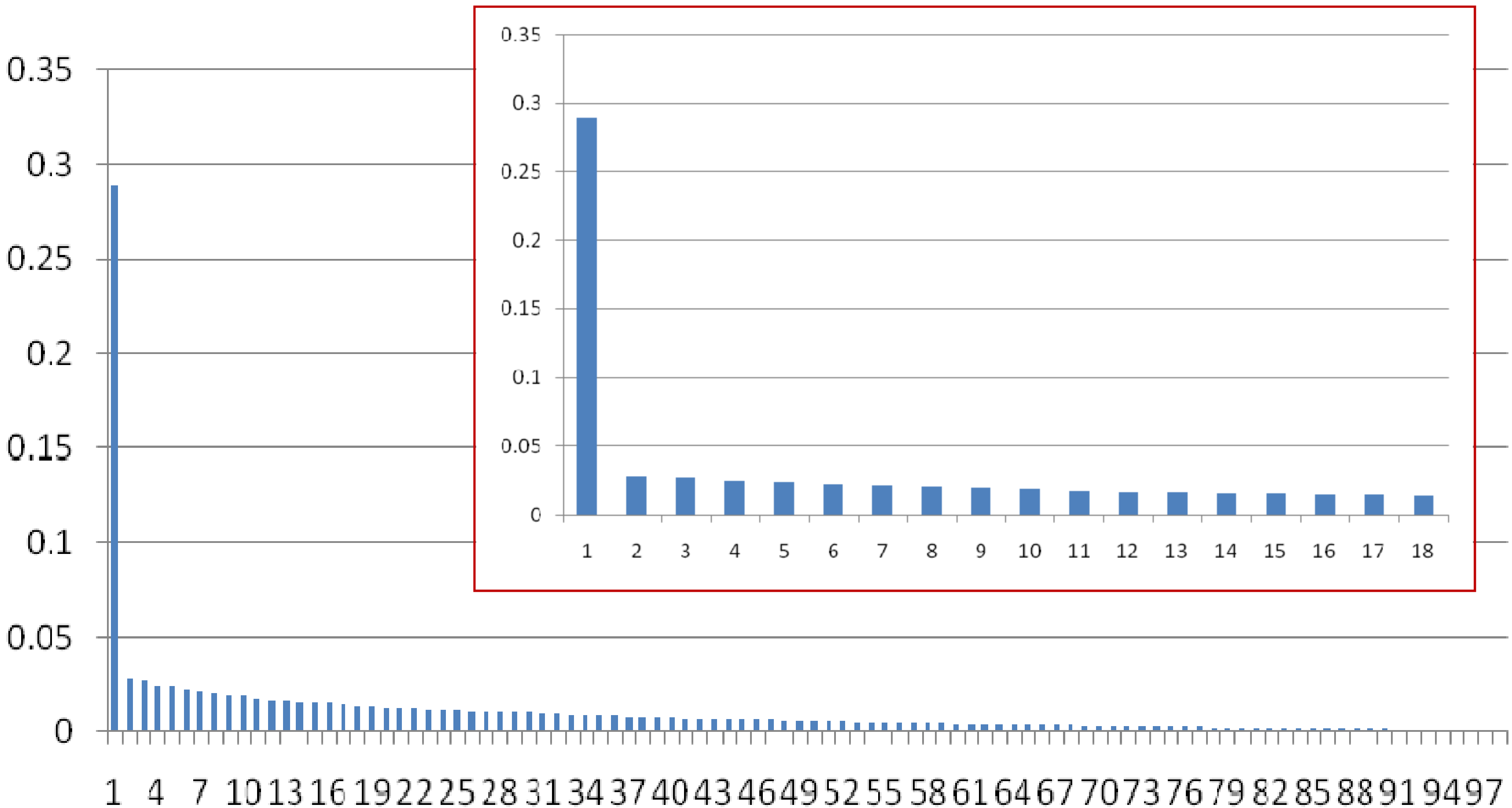
PCA Calculations

-- There are 98 stocks (implied volatilities)

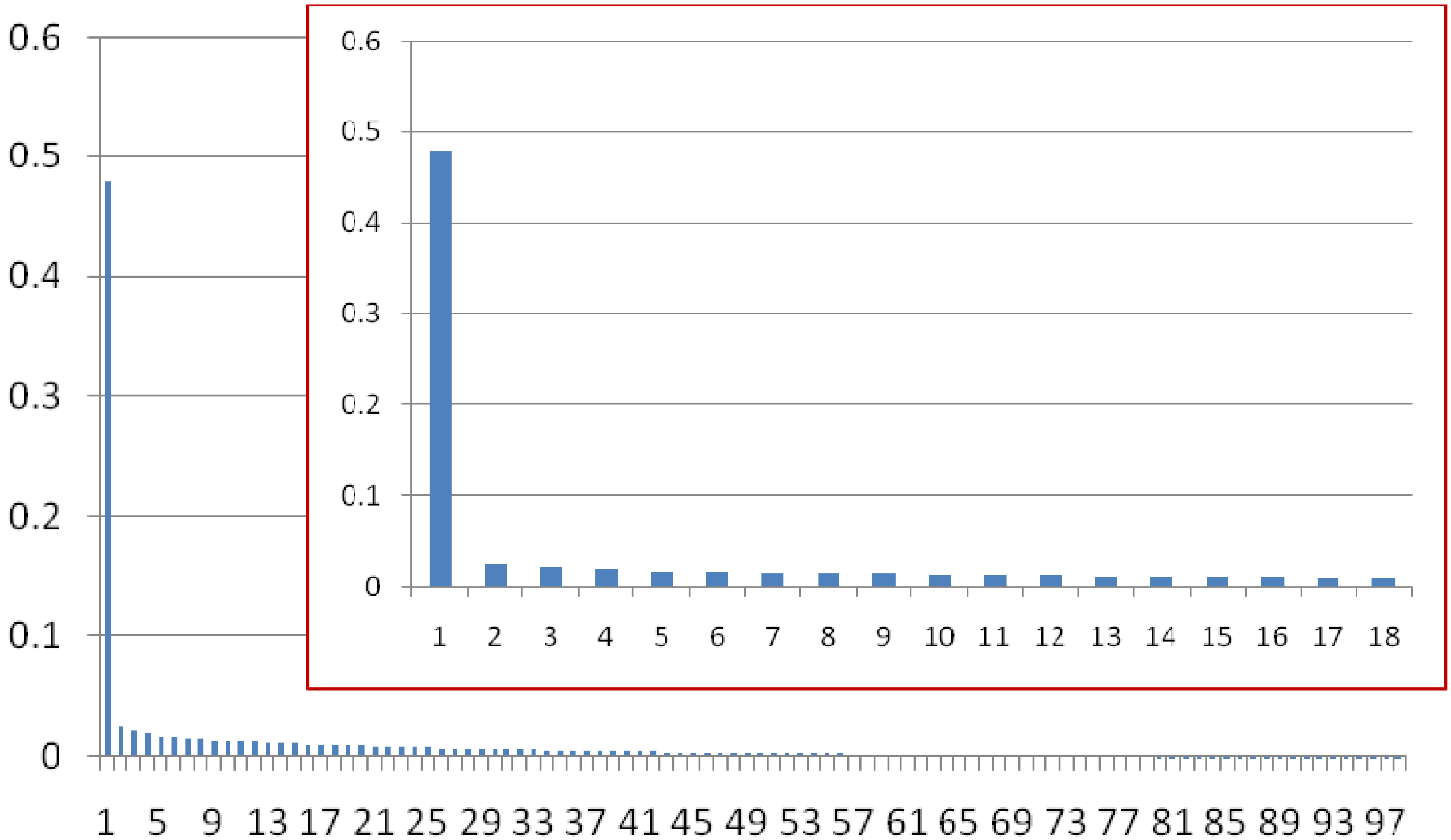
-- We perform a dynamic PCA with window of 180 days

-- 365 successive calculations (spectrum, eigenvectors)

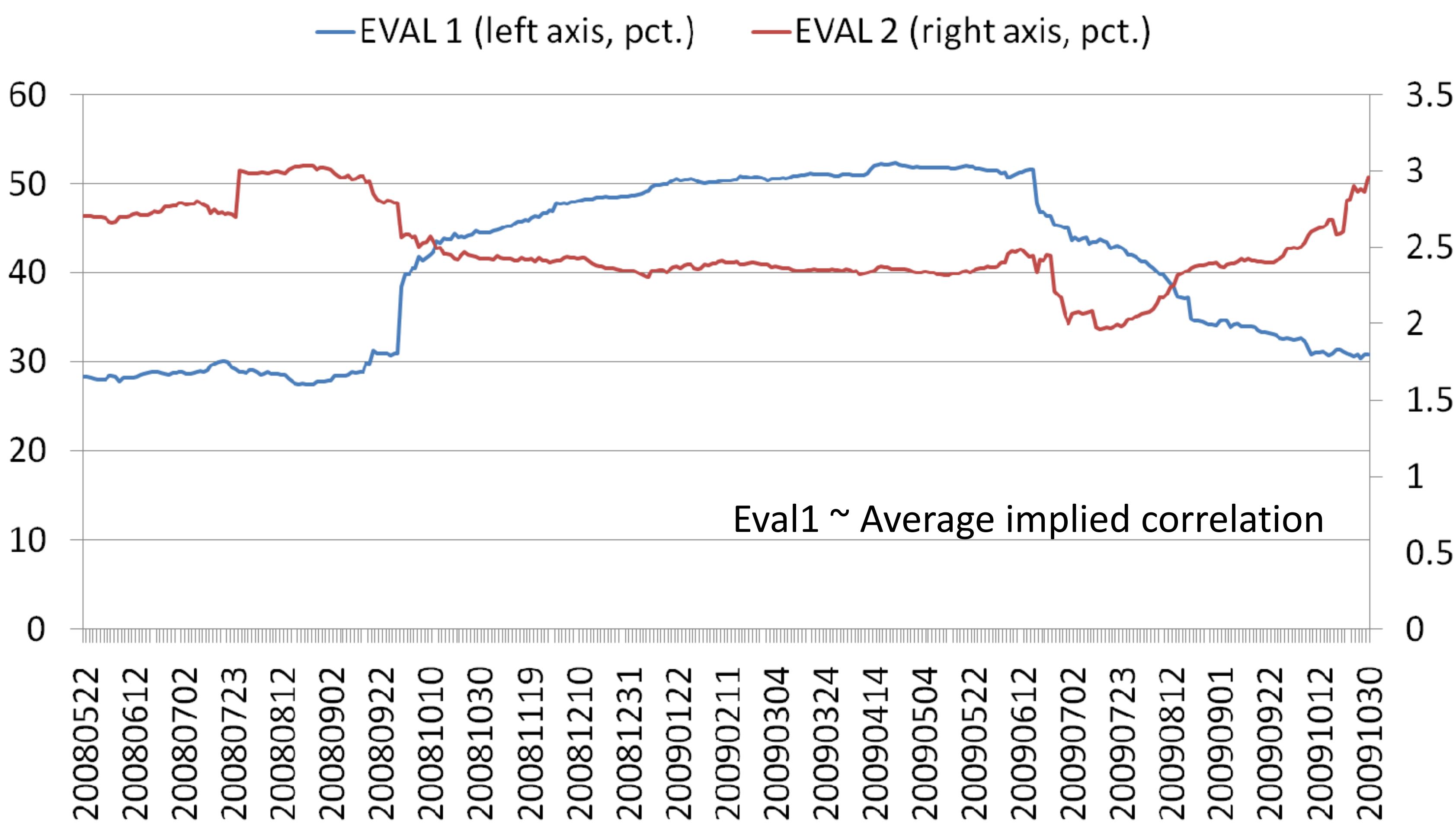
Spectrum on 5/22/2008



Eigenvalues on 12/1/2008



Evolution of 1st and 2nd eigenvalues from May 2008 to Oct 2009



Factor Model

$$\frac{d\sigma_{ATM,i}}{\sigma_{ATM,i}} = \kappa_i \left(\sum_{k=1}^{m} \gamma_{i,k} F_k + \sqrt{1 - \sum_{i=1}^{m} \gamma_{i,k}^2} \; G_k \right)$$

$$\frac{d\sigma_i(x)}{\sigma_i(x)} = \frac{d\sigma_{ATM,i}}{\sigma_{ATM,i}} + \delta_i dx \qquad x = \ln\left(\frac{K}{S}\right), \; dx = -\frac{dS}{S}$$

The motivation for the second equation is that we assume a parametric skew model

$$\sigma(x) = \sigma_{ATM}\left(1 + \delta x + \gamma x^2 + \ldots\right)$$

Alternative Approach using ETFs

$$\frac{d\sigma_i}{\sigma_i} = \beta_i \frac{dS_i}{S_i} + \gamma_i \frac{d\sigma_{ETF(i)}}{\sigma_{ETF(i)}} + \varsigma_i ,$$

$$ETF(i) = \quad \text{ETF associated with stock } i$$

Model the ATM volatility returns as a function of the stock return and changes in the volatility of the sector.

Conjecture: there are fewer systematic factors that explain volatility returns than in the case of stock returns. (m<20)

Volatility skew of stocks and volatility skew of indexes

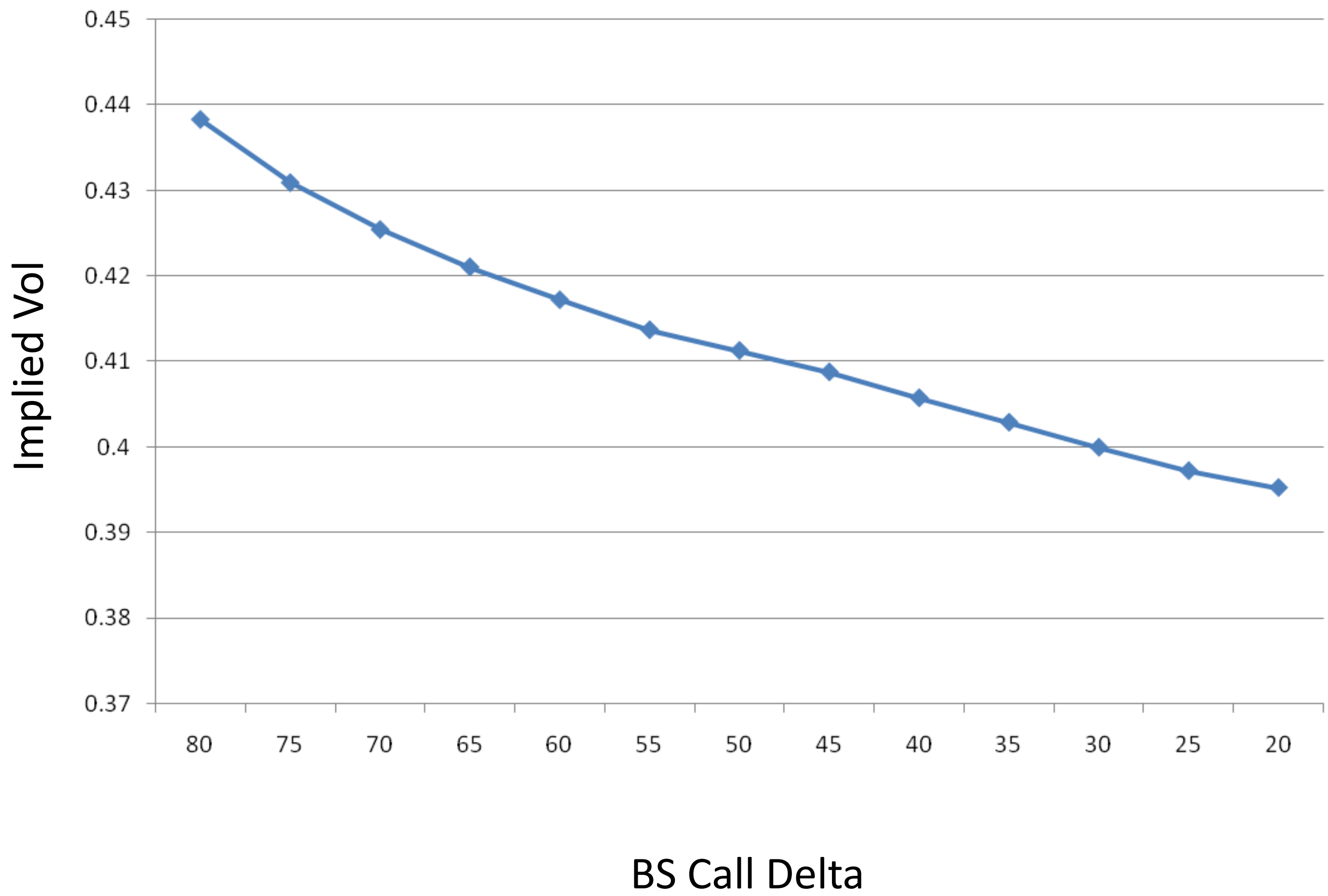
-- For equities, the implied volatility curve is decreasing in the strike price around ATM

-- The effect is more pronounced for indices and ETFs than for single names

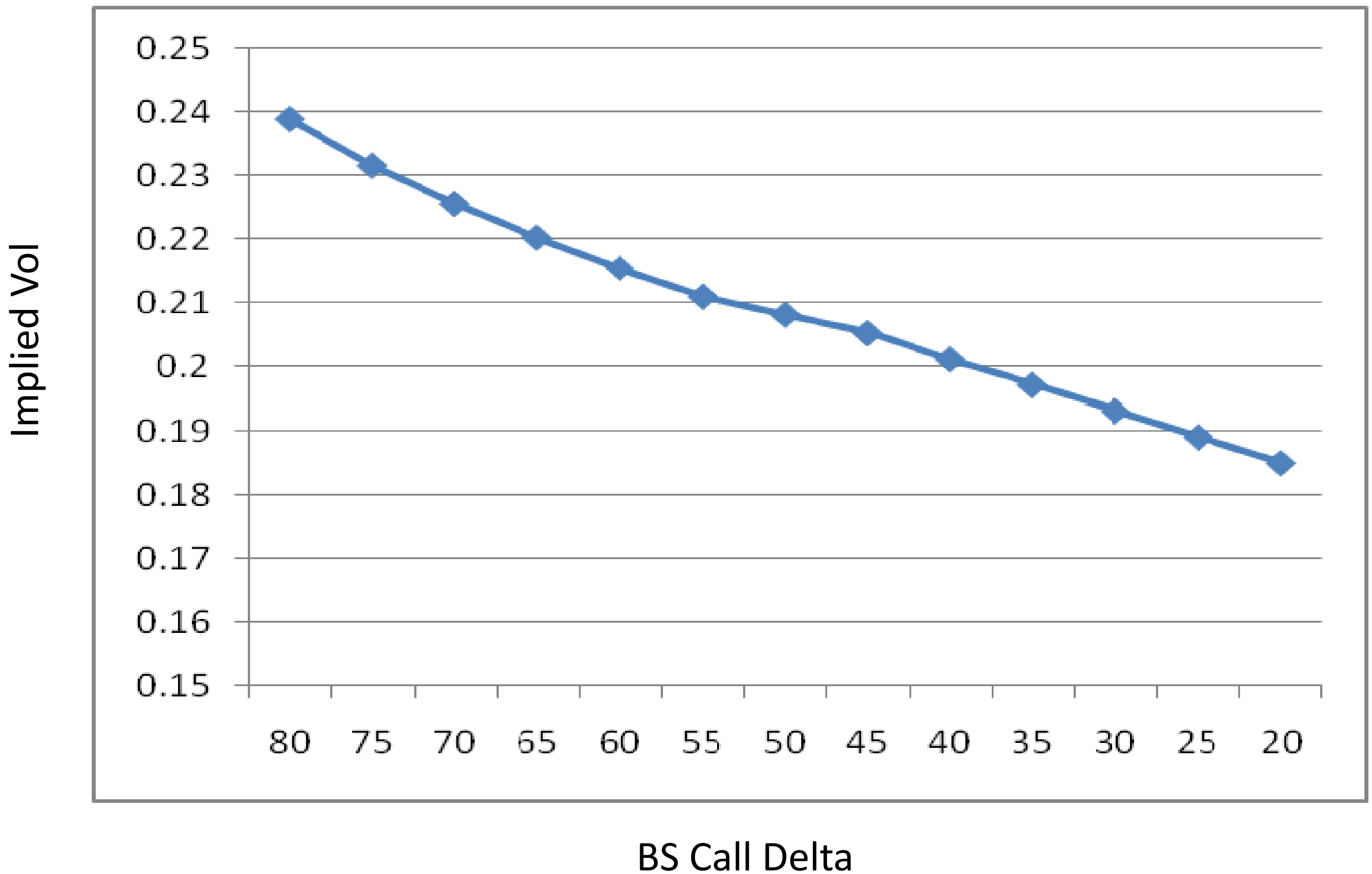
-- Indexes are more skewed than single stocks, presumably due to ``correlation risk''

-- Indexes implied vol curves have less convexity than single-stock implied volatility curves

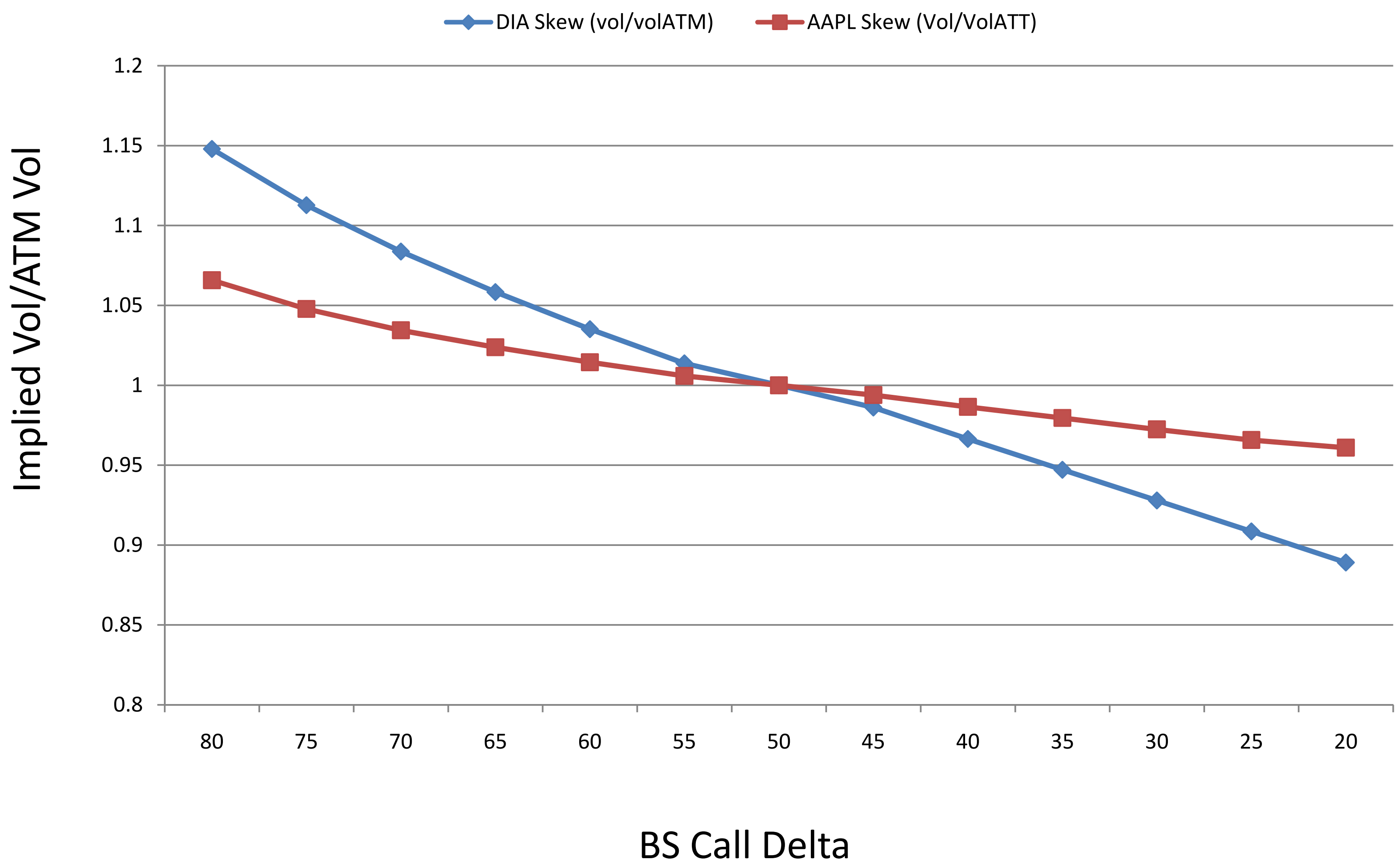
AAPL 30D Vol 9/2/2008



DIA 30D Vol 9/2/2008



AAPL 30D Skew vs. DIA 30D Skew 2/9/2008



Modeling the Volatility Skew

$$x = \ln(K/S)$$

$$\sigma_{imp}(x,t) = \sigma_{imp}(0,t) \cdot \left(1 + \gamma x + \delta x^2 + \ldots\right)$$

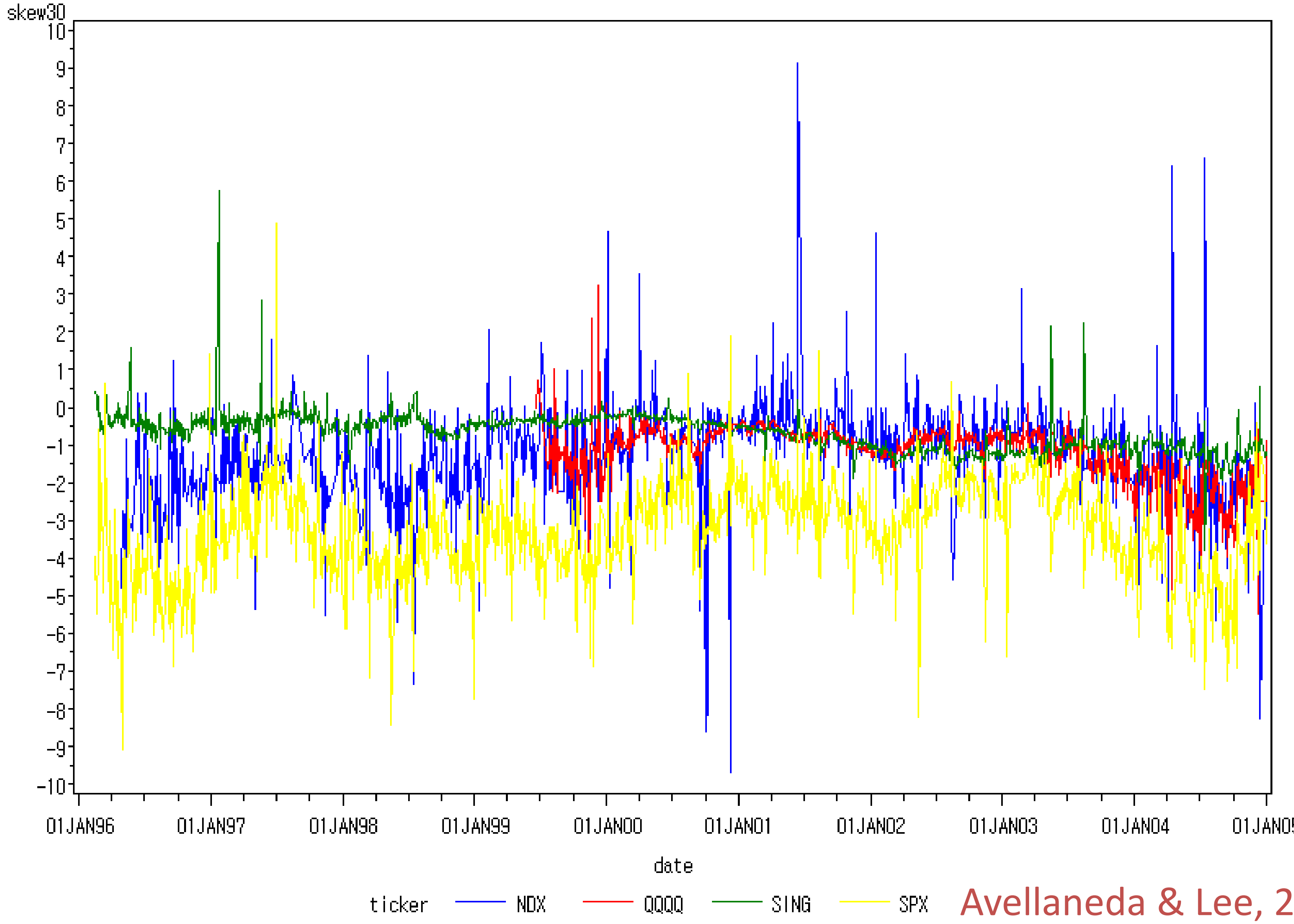
Proposition: Under reasonable assumptions on model (stoch. vol),

If $$\frac{d\sigma_{atm}}{\sigma_{atm}} = \beta \frac{dS}{S} + \varepsilon$$

Then $$\gamma = \frac{\beta}{2}$$

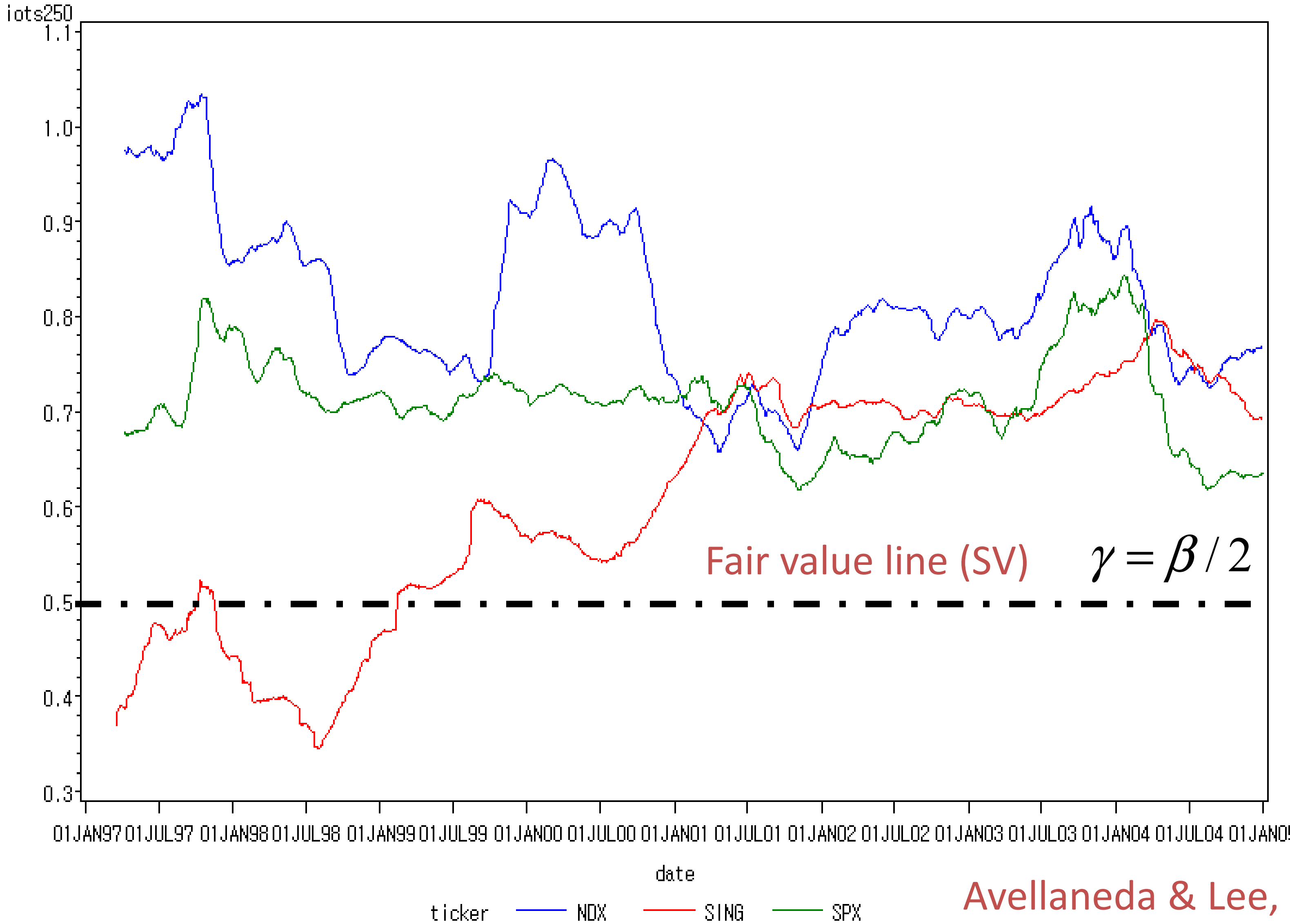
Can also check this directly on data

Evolution of the slope of the 30-day implied volatility curve, 1996-2004



Avellaneda & Lee, 2005

Evolution of ratio [slope/leverage coefficient] The ``roaring 90's''!



Avellaneda & Lee, 2005