Risk and Portfolio Management
Spring 2010

Equity Options: Risk and Portfolio Management

Summary

Review of equity options

Risk-management of options on a single underlying asset

Full pricing versus Greeks

Volatility Surface: PCA

Stress Test (SPAN)

Multi-asset portfolios

Multi-asset option portfolios

Equity Options Markets

- **Single-name options**

  Electronic trading in 6 exchanges, cross-listing of many stocks, penny-wide bid ask spreads for many contracts

- **Index Options**

  S&P 500, NDX, Minis. Traded on the Chicago Mercantile Exchange. VIX options & futures trade in CME as well.

- **ETF Options**

  Most of the large ETFs are optionable. Traded like stocks in multiple exchanges. SPY, QQQQ, XLF are among the most traded options in the US.

Options Markets
Halliburton (HAL) April 09

CALLS								PUTS						
Symbol	**Last**	**Change**	**Bid**	**Ask**	**Volume**	**Open Int**	**Strike**	**Symbol**	**Last**	**Change**	**Bid**	**Ask**	**Volume**	**Open Int**
HALDA.X	12.65	0	11.15	11.3	0	0	5	HALPA.X	0.03	0	N/A	0.04	100	210
HALDU.X	8.5	0	8.65	8.85	2	2	7.5	HALPU.X	0.05	0	0.01	0.06	1	2,237
HALDB.X	5.2	0	6.3	6.35	57	116	10	HALPB.X	0.15	0	0.1	0.12	25	3,775
HALDZ.X	4.2	0.15	4.05	4.15	20	944	12.5	HALPZ.X	0.4	0.12	0.39	0.4	185	10,482
HALDC.X	2.31	0.1	2.3	2.33	220	4,942	15	HALPC.X	1.06	0.33	1.09	1.11	52	10,592
HALDP.X	1.11	0.18	1.09	1.11	495	8,044	17.5	HALPP.X	2.42	0.34	2.36	2.37	196	8,482
HALDD.X	0.43	0.05	0.42	0.44	57	10,693	20	HALPD.X	4.59	0	4.15	4.25	250	12,440
HALDQ.X	0.15	0.02	0.14	0.16	23	7,646	22.5	HALPQ.X	7.25	0	6.4	6.45	25	2,770
HALDE.X	0.05	0.01	0.05	0.06	13	4,060	25	HALPE.X	9.95	0	8.8	8.85	4	1,111
HALDR.X	0.03	0	0.01	0.03	8	5,784	27.5	HALPR.X	12.35	0	11.25	11.35	18	977
HALDF.X	0.01	0	N/A	0.02	20	8,399	30	HALPF.X	14.8	0	13.7	13.9	18	5,772
HALDS.X	0.04	0	N/A	0.04	1	1,698	32.5	HALPS.X	15.5	0	16.2	16.4	20	150
HALDG.X	0.08	0	N/A	0.04	2	1,470	35	HALPG.X	18.93	0	18.7	18.9	5	514
HALDT.X	0.02	0	N/A	0.04	9	604	37.5	HALPT.X	20.59	0	21.2	21.35	40	151
HALDH.X	0.02	0	N/A	0.03	10	1,593	40	HALPH.X	20.6	0	23.7	23.85	10	139
HALDV.X	0.02	0	N/A	0.02	4	2,805	42.5	HALPV.X	26.1	0	26.2	26.4	752	311
HALDI.X	0.02	0	N/A	0.02	1	623	45	HALPI.X	28.6	0	28.7	29	152	0
HALDW.X	0.02	0	N/A	0.02	1	245	47.5	HALPW.X	31.1	0	31.2	31.4	52	13
HALDJ.X	0.02	0	N/A	0.02	7	733	50	HALPJ.X	24.55	0	33.7	33.9	0	0
HALDX.X	0.04	0	N/A	0.02	10	324	52.5	HALPX.X	14.8	0	36.2	36.4	0	0
HALDK.X	0.02	0	N/A	0.02	10	376	55	HALPK.X	19.1	0	38.7	39	0	

HAL= $16.36

Available expirations: Mar09, Apr09, Jul09, Oct09, Jan10, Jan11
2 front months, 2 LEAPS, quarterly cycle (*Jan cycle* for HAL).

Put-Call Parity

$$C - P = Se^{-dT} - Ke^{-rT}$$

Put-call parity holds for American options which are ATM, to within reasonable approximation.

CALLS				PUTS			(C-P+K*(1-r*40/252))/S	d_imp
HALDC.X	2.3	2.33	15	HALPC.X	1.09	1.11	0.988473167	7.26%
HALDP.X	1.09	1.11	17.5	HALPP.X	2.36	2.37	0.989451906	6.65%

Hal pays dividend of 9 cents at the end of Feb, May, Aug, Nov

There are no ex-dividend dates between now and April 20, 2009.

Option markets give an implied cost of carry for the stock (implied forward price), which may be different from the nominal cost of carry. This is due to stock-loan considerations.

DIA Options Apr 18, 2009

Symbol	Last	Change	Bid	Ask	Volume	OpenInt	STRIKE	Symbol	Last	Change	Bid	Ask	Volume	Open Int
DIHDX.X	N/A	0	18.1	18.2	0	0	50	DIHPX.X	0.37	0	0.15	0.19	18	245
DIHDY.X	21	0	17.3	17.4	2	2	51	DIHPY.X	0.39	0	0.17	0.22	105	370
DIHDZ.X	16.3	0	16.3	16.4	1	93	52	DIHPZ.X	0.26	0.22	0.23	0.26	7	225
DIHDA.X	N/A	0	15.45	15.55	0	0	53	DIHPA.X	0.32	0.26	0.28	0.31	5	68
DIHDB.X	N/A	0	14.25	14.35	0	0	54	DIHPB.X	0.4	0.24	0.34	0.37	4	392
DIHDC.X	11.94	0	13.45	13.55	4	14	55	DIHPC.X	0.42	0.38	0.41	0.44	25	765
DIHDD.X	12.35	0.17	12.55	12.65	40	22	56	DIHPD.X	0.51	0.46	0.49	0.52	20	870
DIHDE.X	10.3	0.47	11.6	11.75	10	48	57	DIHPE.X	0.61	0.53	0.59	0.62	72	414
DIHDF.X	8.6	0	10.75	10.85	2	202	58	DIHPF.X	0.73	0.53	0.71	0.73	32	689
DIHDG.X	8.4	0	9.85	9.95	33	211	59	DIHPG.X	0.86	0.54	0.83	0.87	18	658
DIHDH.X	8.4	1.35	9	9.1	48	206	60	DIHPH.X	1	0.75	1	1.02	165	11,734
DIJDI.X	7.7	1.22	8.15	8.3	1	162	61	DIJPI.X	1.21	0.75	1.17	1.2	61	510
DIJDJ.X	7.2	0.8	7.4	7.45	34	228	62	DIJPJ.X	1.43	0.9	1.38	1.4	41	916
DIJDK.X	6.7	1.65	6.6	6.7	137	282	63	DIJPK.X	1.65	0.94	1.61	1.63	108	1,347
DIJDL.X	6	1.6	5.9	5.95	60	444	64	DIJPL.X	1.93	1.03	1.89	1.91	305	1,138
DIJDM.X	5.25	1.41	5.2	5.25	102	825	65	DIJPM.X	2.27	1.18	2.19	2.21	583	1,735
DIJDN.X	4.55	1.32	4.5	4.6	69	1,142	66	DIJPN.X	2.64	1.21	2.52	2.56	213	1,919
DIJDO.X	3.96	1.25	3.9	4	134	945	67	DIJPO.X	3.05	1.4	2.91	2.95	450	2,115
DIJDP.X	3.4	1.08	3.35	3.4	343	1,788	68	DIJPP.X	3.46	1.44	3.3	3.4	217	2,505
DIJDQ.X	2.85	0.91	2.84	2.87	168	1,709	69	DIJPQ.X	3.8	1.85	3.8	3.9	116	1,688
DIJDR.X	2.41	0.82	2.37	2.4	399	9,896	70	DIJPR.X	4.54	1.61	4.35	4.4	144	2,829
DIJDS.X	1.92	0.64	1.94	1.98	117	1,465	71	DIJPS.X	5.14	1.86	4.9	5	51	3,035
DIJDT.X	1.58	0.58	1.57	1.6	262	1,998	72	DIJPT.X	5.6	2.2	5.55	5.65	7	2,528
DIJDU.X	1.27	0.5	1.25	1.29	215	1,924	73	DIJPU.X	6.28	2.37	6.2	6.35	22	1,580
DIJDV.X	1	0.4	0.99	1.02	235	1,761	74	DIJPV.X	7.1	2.05	6.95	7.05	2	1,253
DIJDW.X	0.78	0.3	0.77	0.79	182	3,421	75	DIJPW.X	7.8	2.28	7.75	7.85	29	1,292
DIJDX.X	0.6	0.16	0.58	0.61	26	2,652	76	DIJPX.X	10.3	0	8.55	8.65	29	1,008
DIJDY.X	0.44	0.14	0.44	0.47	27	2,055	77	DIJPY.X	9.5	2.36	9.4	9.5	5	943
DIJDZ.X	0.32	0.05	0.32	0.35	81	1,800	78	DIJPZ.X	10.65	0.75	10.3	10.4	4	1,290
DIJDA.X	0.26	0.09	0.24	0.26	140	1,147	79	DIJPA.X	11.83	1.37	11.2	11.3	3	1,006
DIJDB.X	0.19	0.08	0.17	0.2	48	8,568	80	DIJPB.X	13.57	1.29	12.15	12.25	3	1,352
DIJDC.X	0.11	0	0.12	0.15	9	3,494	81	DIJPC.X	15.13	0	13.1	13.2	26	5,989
DAVDD.X	0.1	0	0.09	0.12	92	2,455	82	DAVPD.X	16.6	0	14.3	14.45	10	1,184
DAVDE.X	0.07	0.01	0.06	0.09	3	3,218	83	DAVPE.X	16.44	1.22	15.3	15.4	1	1,016
DAVDF.X	0.05	0	0.05	0.08	23	1,470	84	DAVPF.X	16.85	1.28	16.3	16.4	3	843
DAVDG.X	0.04	0	0.03	0.07	11	4,203	85	DAVPG.X	17.2	1.55	17.3	17.4	30	496
DAVDH.X	0.02	0	0.02	0.06	3	841	86	DAVPH.X	17.7	0	18.25	18.4	1	91
DAVDI.X	0.04	0	N/A	0.05	10	617	87	DAVPI.X	21.78	0	19.25	19.35	3	305
DAVDJ.X	0.04	0	N/A	0.05	8	748	88	DAVPJ.X	19.5	0	20.25	20.35	10	124
DAVDK.X	0.04	0.01	N/A	0.04	30	450	89	DAVPK.X	15.9	0	21.25	21.35	15	56
DAVDL.X	0.04	0	N/A	0.04	30	927	90	DAVPL.X	16.95	0	22.2	22.35	5	58
DAVDM.X	0.03	0	N/A	0.04	4	787	91	DAVPM.X	17.5	0	23.2	23.35	2	78

Implied Dividend Yield for DIA
April 18, 2009 Options

CALLS				PUTS			(C-P+K*(1-r*40/252))/S	d_imp
DIJDP.X	3.35	3.4	68	DIJPP.X	3.3	3.4	0.995267636	2.98%
DIJDQ.X	2.84	2.87	69	DIJPQ.X	3.8	3.9	0.994951292	3.18%

Dividend Yield from Yahoo.com= 3.30%

Actual payments are approx 15 cents / month ~ $1.80 ~ 2.60%

Step1 in understanding options markets: find the implied dividend from the market.

If the implied dividend is different from the nominal dividend then

-- check for HTB if $d_{imp} > d_{nom}$

-- check for dividend reductions if $d_{imp} < d_{nom}$

Calculation of d_{nom}, d_{imp}

$$d_{nom} = \frac{-1}{T} \ln\left(\frac{S - \sum_{i=1}^{n} D_i e^{-rT_i}}{S} \right)$$

Dividend payment dates

$$d_{imp} = \frac{-1}{T} \ln\left(\frac{C_{atm} - P_{atm} + K_{atm} e^{-rT}}{S} \right)$$

LDK Solar Co. (LDK) May 2010 options series

Pricing Date	3/23/2010	Rate	0.12%	Spot	6.9
Expiration	5/22/2010			Days	44

CALLS							PUTS						
Symbol	**Last**	**Bid**	**Ask**	**Volume**	**Open Int**	**Strike**	**Symbol**	**Last**	**Bid**	**Ask**	**Volume**	**Open Int**	**idiv**
DLO100522C00 005000	N/A	1.9	2	0	0	5	DLO100522 P00005000	0.21	0.2	0.3	60	26	15%
DLO100522C00 006000	N/A	1.1	1.3	0	0	6	DLO100522 P00006000	0.6	0.5	0.6	30	30	15%
DLO100522C00 007000	0.65	0.7	0.7	175	73	7	DLO100522 P00007000	N/A	1	1.1	0	0	17%
DLO100522C00 008000	0.35	0.3	0.35	40	206	8	DLO100522 P00008000	N/A	1.7	1.9	0	0	28%
DLO100522C00 009000	0.15	0.2	0.2	9	101	9	DLO100522 P00009000	N/A	2.5	2.8	0	0	26%

LDK is a hard-to-borrow stock with repo rate of approximately -12.5% in one of the brokers. No ``real'' dividend is paid.

Choosing the dividend for implied volatility calculations

Since the dividend is an attribute of the stock and not of the options, we must a constant dividend per maturity to fit all option prices irrespective of the strike.

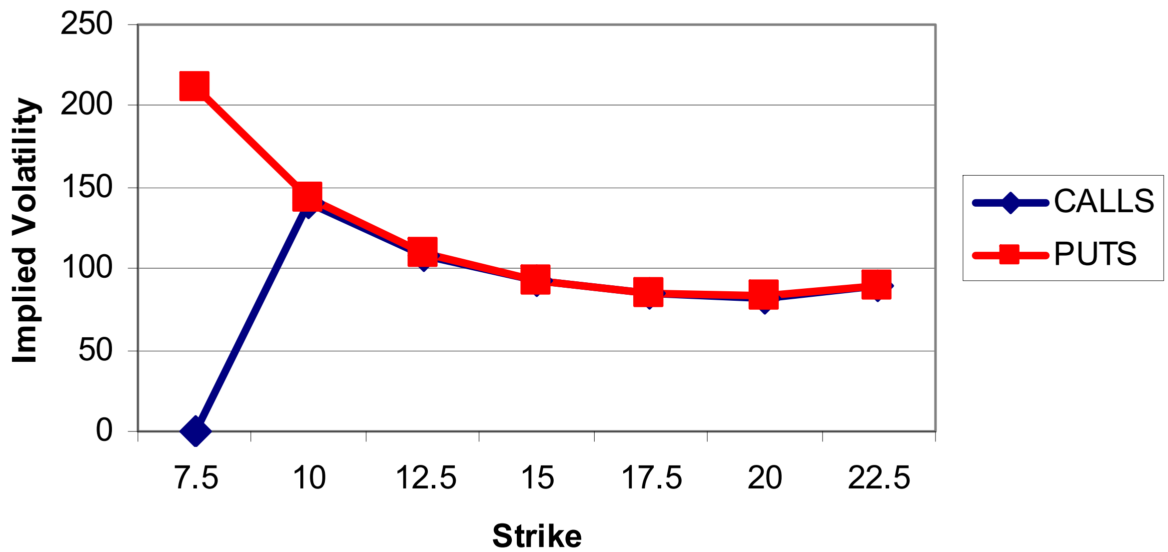
Based on this choice of dividend, we can then calculate the implied volatility of each contract and construct the implied volatility curves for the options in the given maturity.

The market convention is to use the mid-market NBBO for puts and calls, the current rate (FF) and the implied dividend to calculate implied volatilities.

Note: implied dividends for different strike form an increasing curve always in the case of HTB stocks (Avellaneda and Lipkin, *RISK*, 2009)

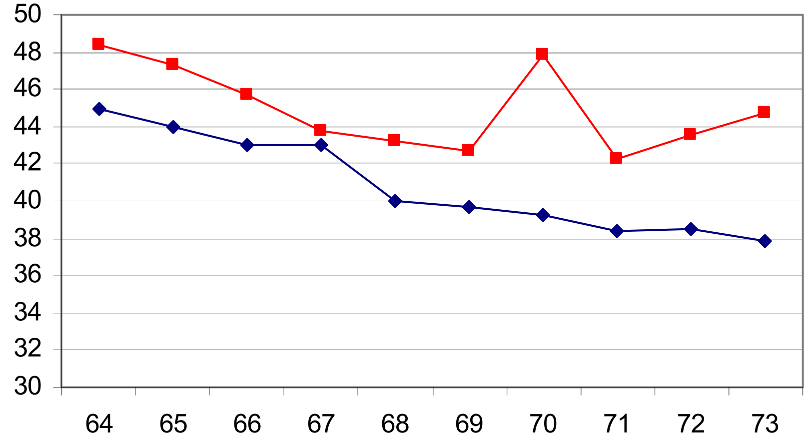
Implied Volatility
HAL April 09

CALLS							PUTS					
Symbol	Last	Bid	Ask	IVOL	Delta	Strike	Symbol	Last	Bid	Ask	IVOL	Delta
HALDU.X	8.5	8.65	8.85	na	1.00	7.5	HALPU.X	0.05	0.01	0.06	211	0.00
HALDB.X	5.2	6.3	6.35	141	0.99	10	HALPB.X	0.15	0.1	0.12	144	-0.01
HALDZ.X	4.2	4.05	4.15	108	0.94	12.5	HALPZ.X	0.4	0.39	0.4	109	-0.05
HALDC.X	2.31	2.3	2.33	92.4	0.76	15	HALPC.X	1.06	1.09	1.11	93	-0.24
HALDP.X	1.11	1.09	1.11	85.1	0.36	17.5	HALPP.X	2.42	2.36	2.37	85	-0.63
HALDD.X	0.43	0.42	0.44	82.4	0.09	20	HALPD.X	4.59	4.15	4.25	84	-0.90
HALDQ.X	0.15	0.14	0.16	89.3	0.02	22.5	HALPQ.X	7.25	6.4	6.45	90	-0.97

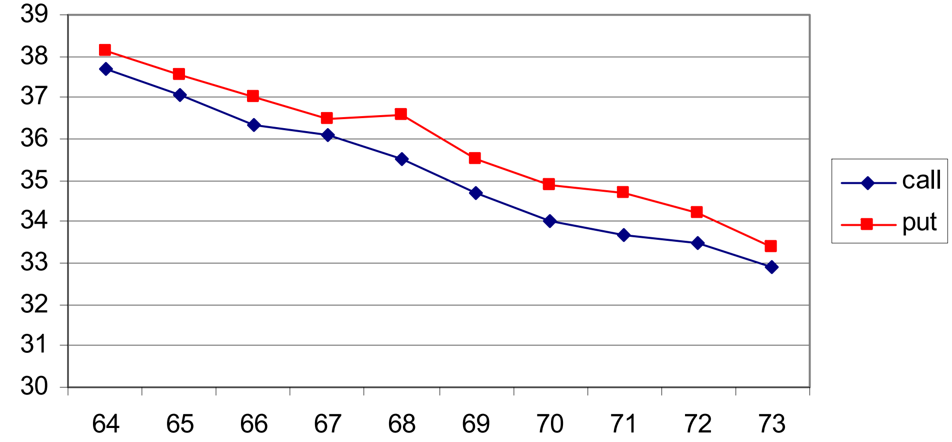


DIA Volatility Surface, March 10 2009, 12:00 noon

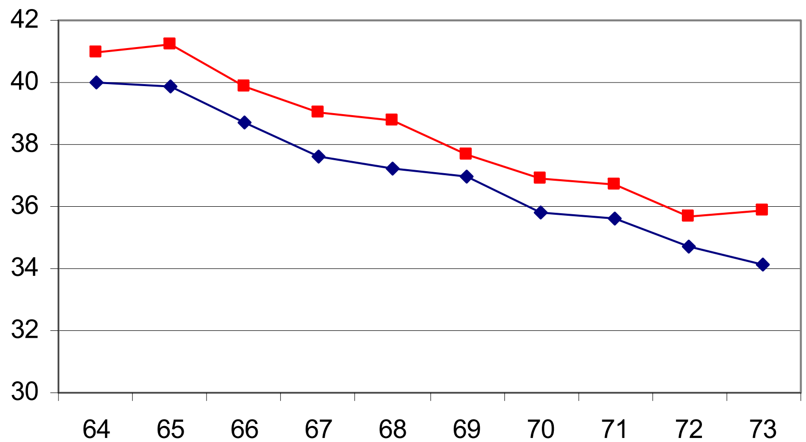
DIA, Mar09



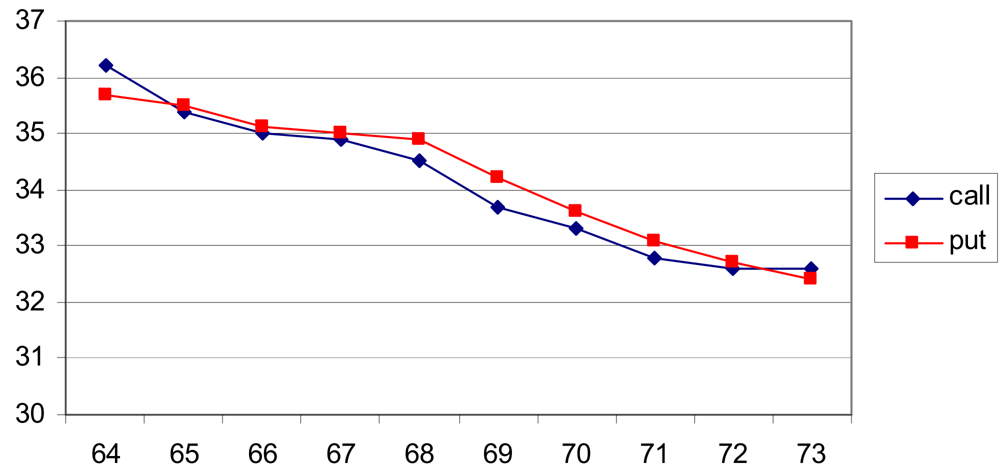
DIA, Jun 30, 09



DIA, Apr09



DIA, Sep 30, 09



These curves move in time.

Modeling the Volatility Risk

1. Compute the historical volatility of a constant maturity series by interpolation over fixed maturities.
   (Typically, for equities: 30d , 60 d, 90 d, 180 d, etc)

2. Express the implied volatilities in terms of moneyness or deltas. Deltas is better because this takes into account the volatility of the underlying asset as well.

3. Study the variations of the implied volatility curve for each maturity using PCA & extreme-value theory (Student T)

4. Deduce a model for the variation of implied volatilities for portfolio risk analysis

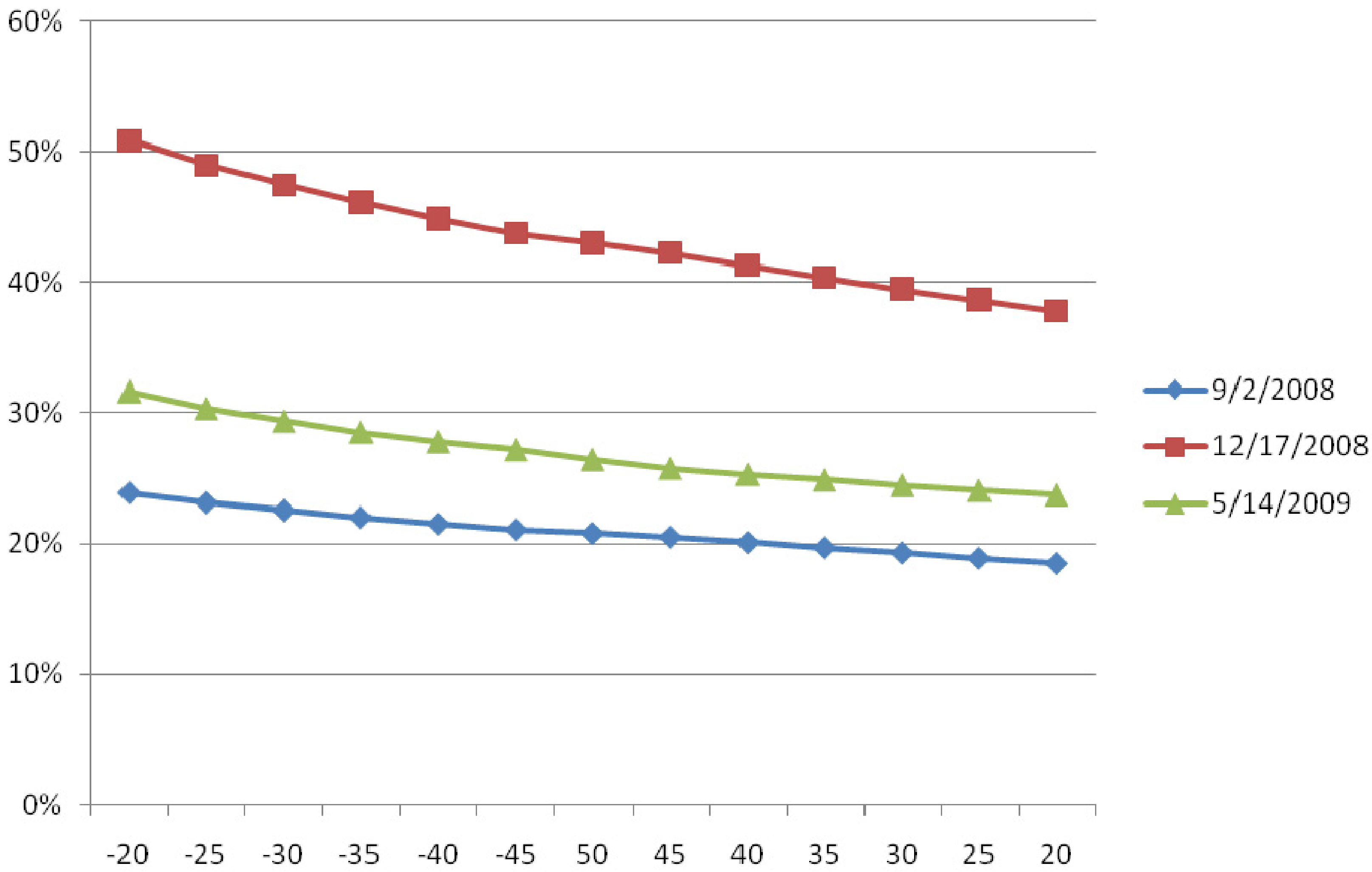
The Data (example with DIA)

OTM Puts | OTM Calls

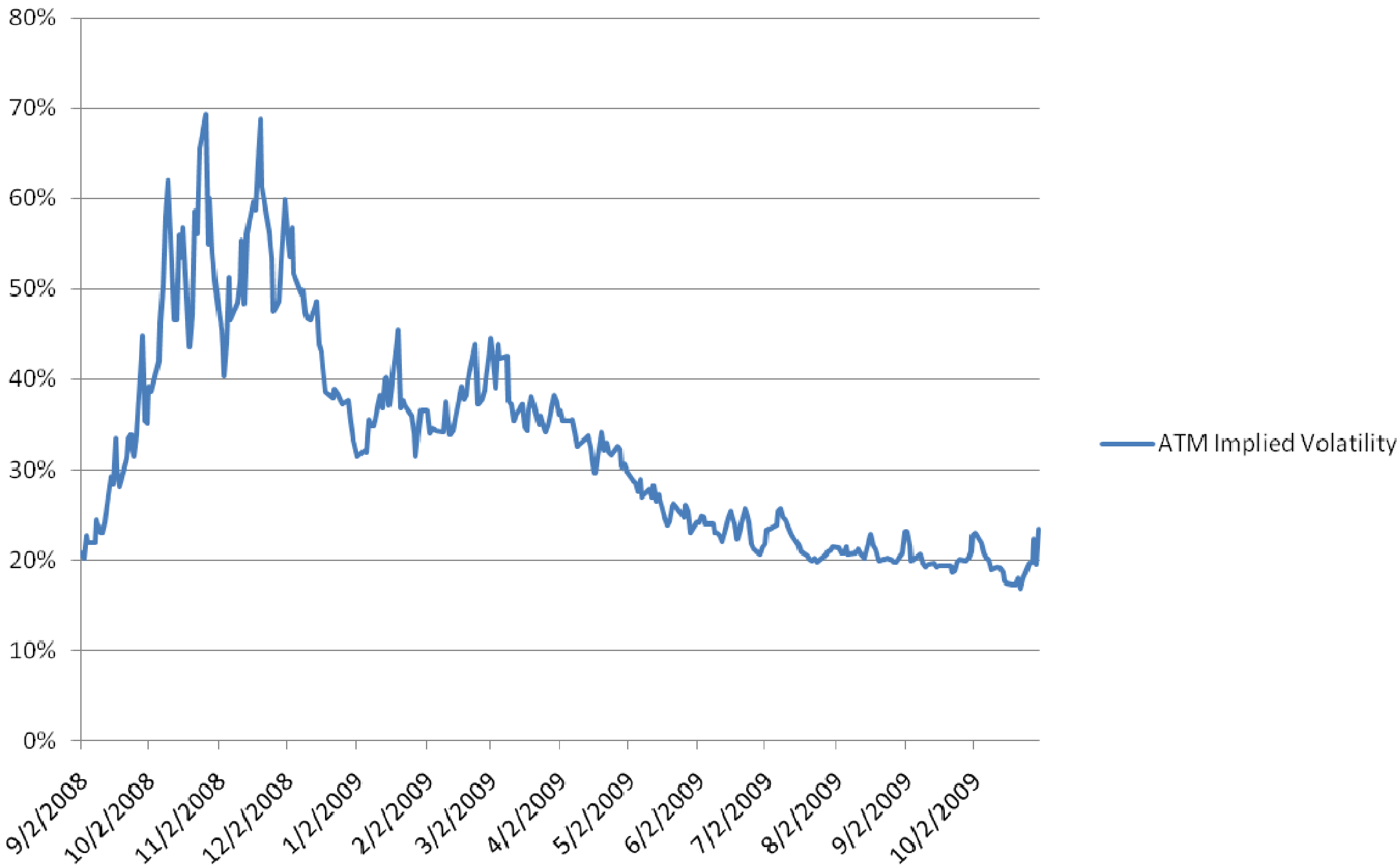
date\delta	-20	-25	-30	-35	-40	-45	50	45	40	35	30	25	20
9/2/2008	23.9%	23.2%	22.6%	22.0%	21.5%	21.1%	20.8%	20.5%	20.1%	19.7%	19.3%	18.9%	18.5%
9/3/2008	23.1%	22.4%	21.9%	21.3%	20.9%	20.4%	20.2%	20.1%	19.7%	19.3%	18.9%	18.5%	18.1%
9/4/2008	26.2%	25.6%	25.0%	24.6%	24.2%	23.8%	22.7%	21.6%	21.3%	21.0%	20.7%	20.4%	20.0%
9/5/2008	25.0%	24.3%	23.7%	23.2%	22.8%	22.3%	21.9%	21.5%	21.1%	20.7%	20.4%	20.0%	19.6%
9/8/2008	24.9%	24.2%	23.6%	23.0%	22.5%	22.0%	21.9%	21.7%	21.3%	20.8%	20.4%	19.9%	19.5%

We consider data from 9/2/2008 until 10/30/2009, organized by Deltas (13 strikes per day)

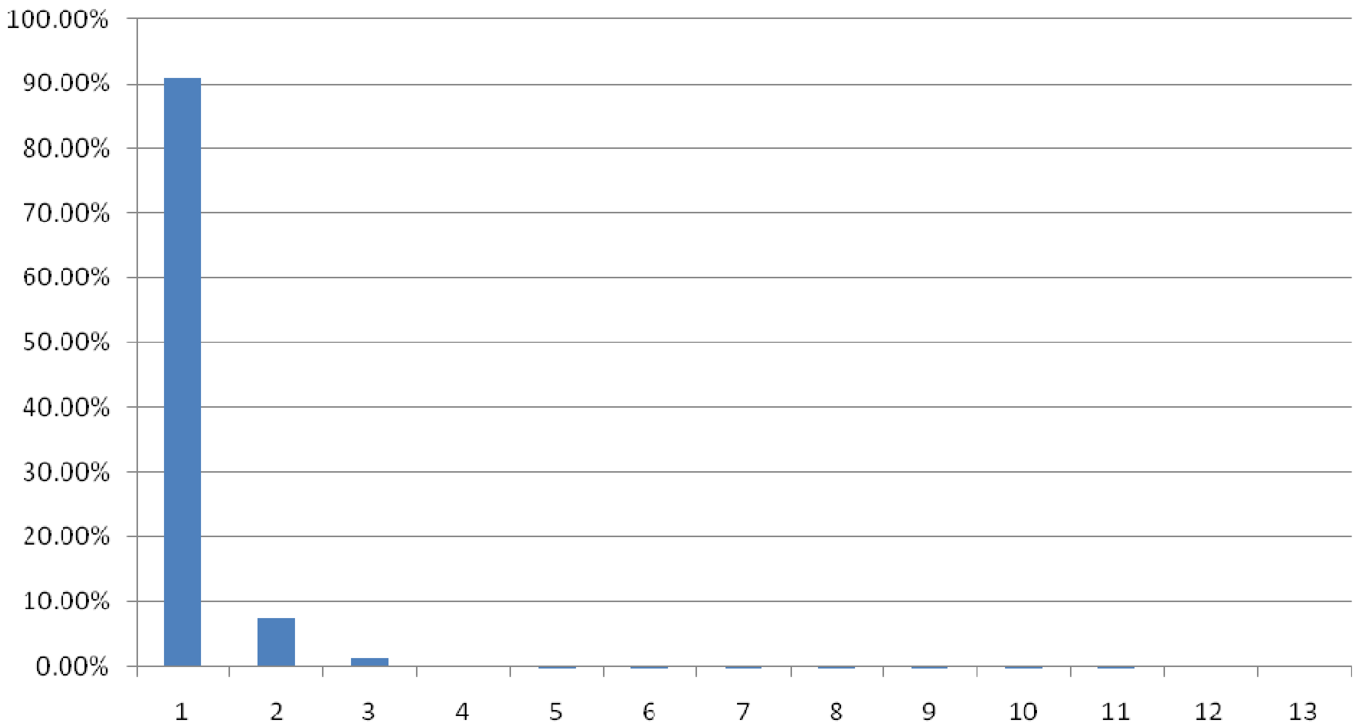
DIA 30 day Implied Vol Curves



DIA ATM Volatility
Sep 2, 2008 – Oct 30 2009



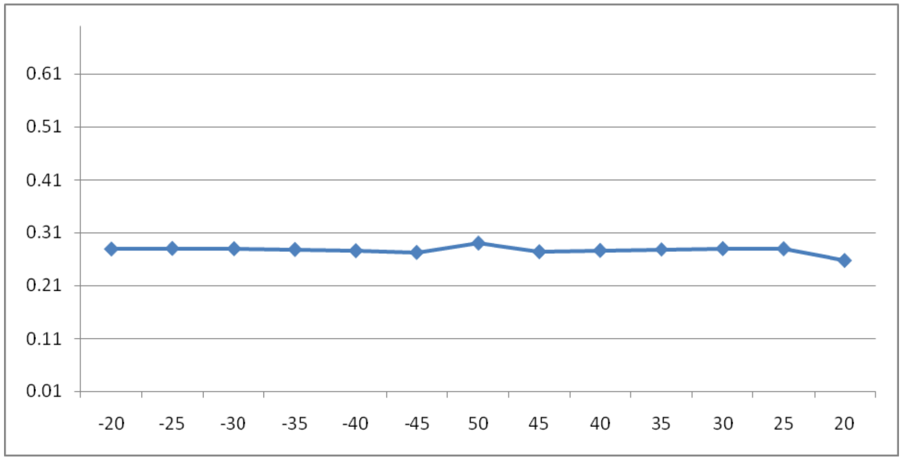
Eigenvalues of the Correlation Matrix
30 Day Ivol returns



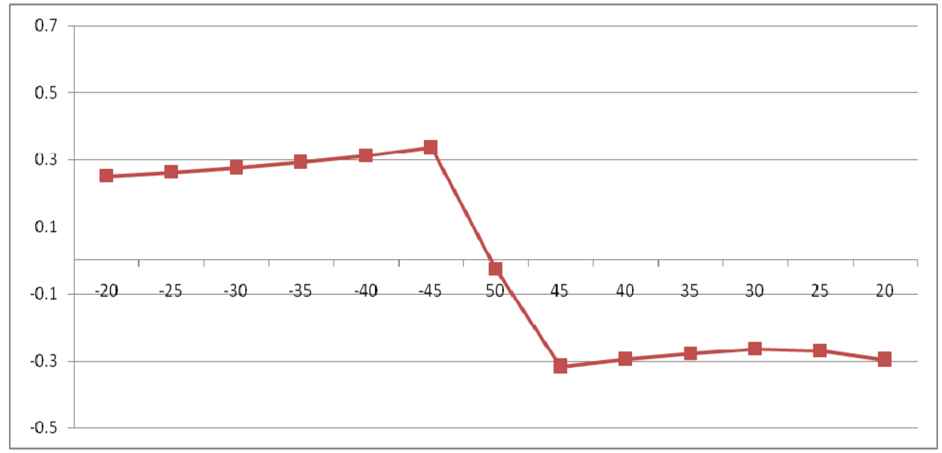
EIGENVALUE
90.91%
7.51%
1.28%
0.27%
0.01%
0.01%
0.00%
0.00%
0.00%
0.00%
0.00%
0.00%
0.00%

Eigenvectors and their explanatory power

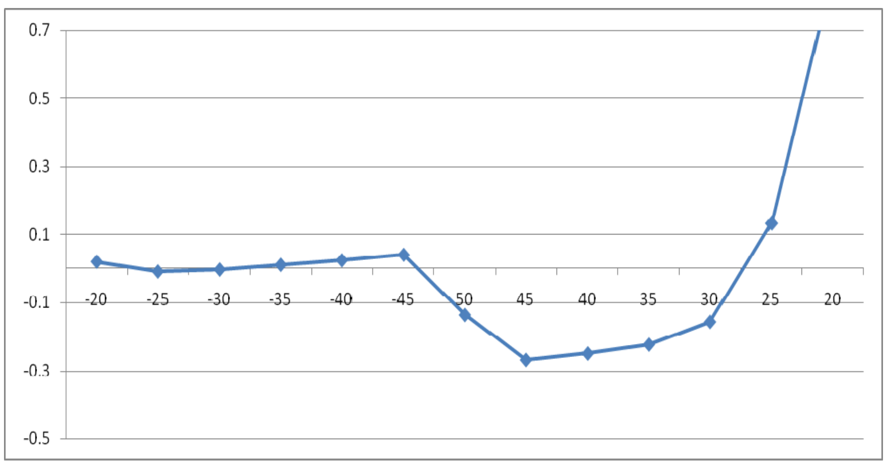
1st Eigenvector 91.1%



2nd Eigenvector 7.51%



3rd Eigenvector 1.28%



Most of the risk is in the parallel shift, i.e. exposure to the ATM vol

The second EV corresponds to the classical skew, i.e. exposure to risk-reversals.

RR= long 30 D put / short 30 D call

Risk-model for single-name option portfolios

$$R_{\sigma(\Delta)} = \beta_1 R_1 + \beta_2 R_2 \left(\frac{\Delta_c - 50}{50} \right) + \varepsilon$$

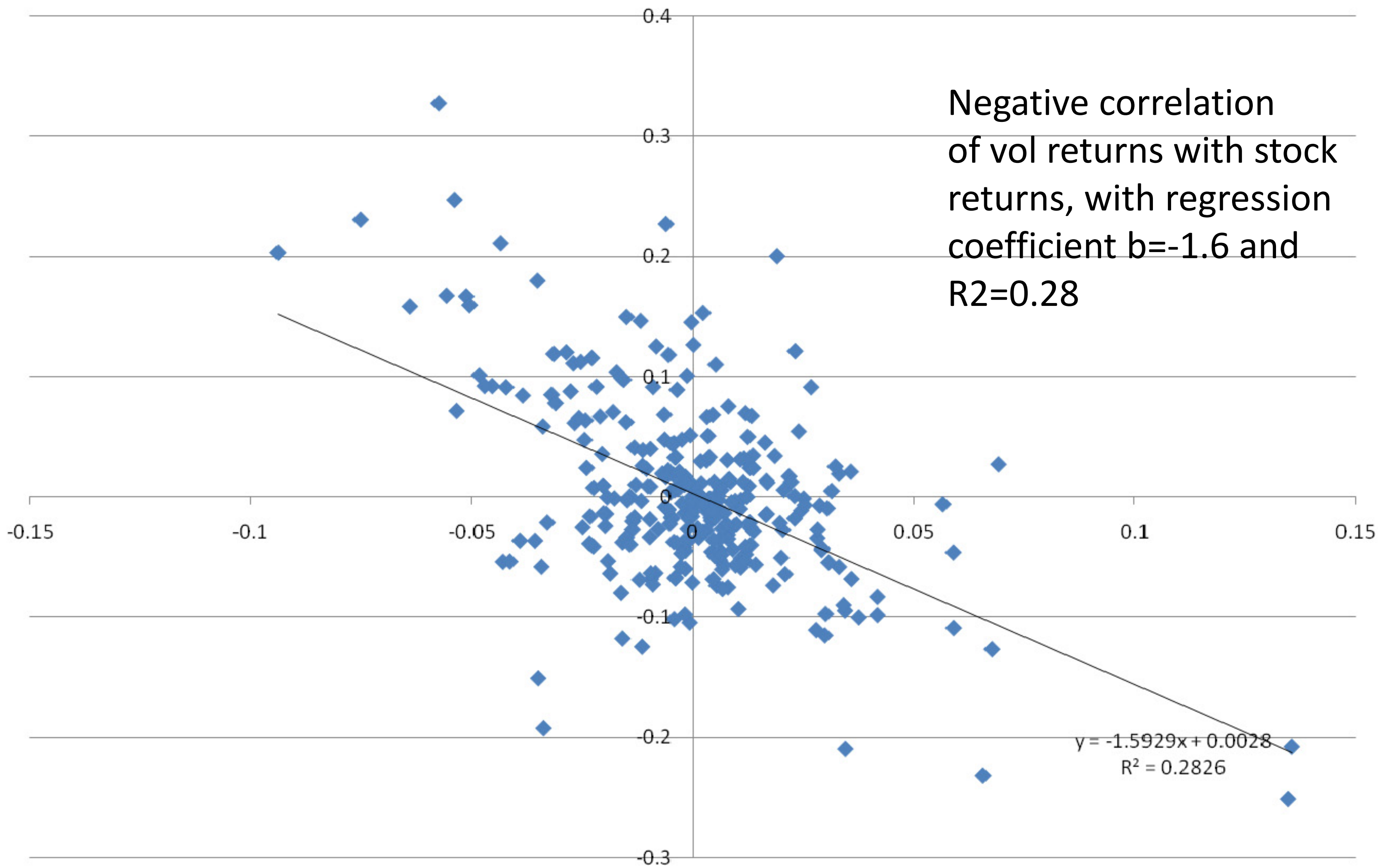
or

$$\frac{d\sigma(\Delta)}{\sigma(\Delta)} = \beta_1 \frac{d\sigma_{atm}}{\sigma_{atm}} + \beta_2 \left(\frac{\Delta_c - 50}{50} \right) R_2 + \varepsilon$$

The distributions for ATM vol returns and RR returns can be estimated from historical data.

One important consideration: ATM vol is negatively correlated to stock prices, so there is a further analysis needed to specify the joint distribution of stocks and volatility

X=DIA returns, Y=ATM vol returns



Negative correlation of vol returns with stock returns, with regression coefficient b=-1.6 and R2=0.28

Coupled model for stock and vol shocks

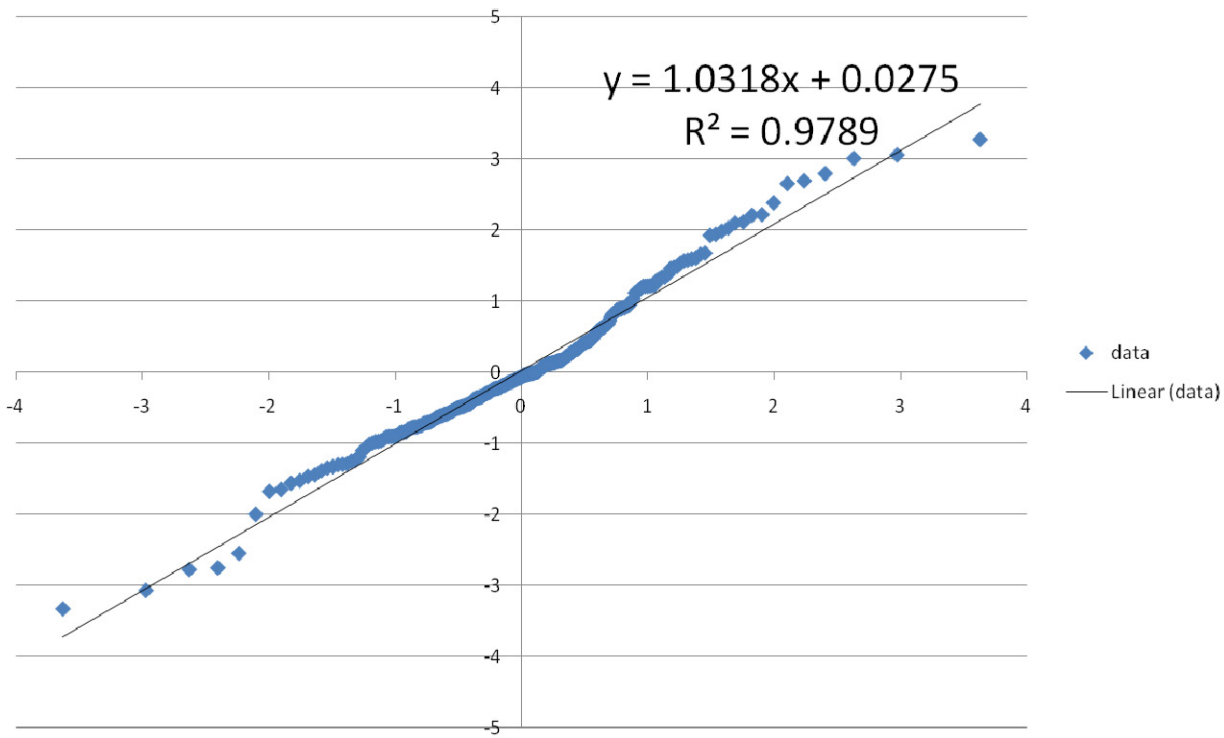
$$R_{\sigma(\Delta)} = \beta_1 R_1 + \beta_2 R_2 \left(\frac{\Delta_c - 50}{50} \right) + \varepsilon$$

$$\frac{d\sigma(\Delta)}{\sigma(\Delta)} = \beta_1 \frac{d\sigma_{atm}}{\sigma_{atm}} + \beta_2 \left(\frac{\Delta_c - 50}{50} \right) R_2 + \varepsilon$$

$$\boxed{R_{\sigma(\Delta)} = \beta_1 (\gamma_1 R_s + \gamma_2 E) + \beta_2 R_2 \left(\frac{\Delta_c - 50}{50} \right)}$$

Stock return | Idiosyncratic vol return | RR return

Extreme-value analysis: ATM vol



QQ-plot vs. Student T with DF=4

prob	student	data
0.0034	-3.633	-3.333
0.0068	-2.976	-3.074
0.0102	-2.633	-2.779
0.01361	-2.406	-2.755
0.01701	-2.238	-2.55
0.02041	-2.106	-1.999
0.02381	-1.997	-1.678
0.02721	-1.905	-1.651
0.03061	-1.825	-1.561
0.03401	-1.755	-1.526
0.03741	-1.693	-1.468
0.04082	-1.637	-1.444
0.04422	-1.585	-1.385
0.04762	-1.538	-1.347
0.05102	-1.495	-1.328

Left tail vs right tail using DF=4

Extreme down moves

prob	student	data
0.0034	-3.633	-3.333
0.0068	-2.976	-3.074
0.0102	-2.633	-2.779
0.01361	-2.406	-2.755
0.01701	-2.238	-2.55
0.02041	-2.106	-1.999
0.02381	-1.997	-1.678
0.02721	-1.905	-1.651
0.03061	-1.825	-1.561
0.03401	-1.755	-1.526
0.03741	-1.693	-1.468
0.04082	-1.637	-1.444
0.04422	-1.585	-1.385
0.04762	-1.538	-1.347
0.05102	-1.495	-1.328

Extreme up moves moves

prob	student	data
0.9558	1.5853	1.99021
0.9592	1.6366	2.0349
0.9626	1.6929	2.10579
0.966	1.7554	2.11977
0.9694	1.8255	2.21635
0.9728	1.9051	2.22458
0.9762	1.9971	2.39156
0.9796	2.1058	2.66136
0.983	2.2381	2.70045
0.9864	2.406	2.8036
0.9898	2.6331	3.01731
0.9932	2.9757	3.06495
0.9966	3.6328	3.28219

Risk-management of option portfolios

Portfolio change =

$$\sum_{K,T,a=p,c} Q_{K,T,a}\left[BS_a\left(S_0(1+R_s), T-\Delta T, K, r_T, d_T, \sigma_{K,T}(1+R_{\sigma_{K,T}})\right) - BS_a\left(S_0, T, K, r_T, d_T, \sigma_{K,T}\right)\right]$$

where

$Q_{K,T,a}$ = number of options with strike K, maturity T, put or call ($a = p$ or c)

S_0 = stock price

$\sigma_{K,T}$ = implied volatility

Simulate risk-scenarios using the factor model described above and analyze extreme values

Risk scenarios correspond to joint stock shocks and vol shocks $\left(R_s, R_{\sigma_{K,T}}\right)$