Risk and Portfolio Management
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Statistical Methods for

Mortgage-Backed Securities

Statistical Methods for Risk-Management of Agency MBS

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Agency pass-through securities

Agency MBS are pools of residential mortgages which have standardized characteristics (coupon, maturity).

Principal is guaranteed by FN/FD/GN in case of mortgagee default

Agency MBS look like amortizing bonds with a random amortization schedule which is related to the prepayment of the different mortgages

Variables associated with MBS:

CUSIP, WALA, Current Face Value, Actual CPR, Projected CPR, Actual SMM, Projected SMM, Coupon, WAC, TBA Price

Classical approaches consist in building prepayment models to understand the risk in holding MBS (e.g. as collateral).

We believe that a more data-driven approach eliminates model risk and should be used whenever possible, especially in risk-management.

Information available on MBS

CUSIP	identifier
Wala	weighted average life
Current Face	outstanding principal
Actual CPR	monthly Conditional Prepayment Rate (CPR)
Projected CPR	monthly CPR projection
Actual SMM	monthly Single Monthly Mortality
Projected SMM	Monthly SMM
difference in CPR	Actual-projected
difference in SMM	Actual-projected
Coupon	Bond coupon
WAC	weighted average mortgage rate for the pool
Price	Clean price -- tracks closely near-month TBA

TBA= to-be-announced contract (OTC)

TBA

For "vanilla" or "generic" 30-year pools (FN/FG/GN) with coupons of 3.5% - 7%, one can see the prices posted by dealers on a TradeWeb screen called To Be Announced (TBA).

These are forward prices for the next 3 delivery months since pools haven't been ``cut'' — only the issuing agency, coupon and dollar amount are revealed.

A specific pool whose characteristics are known would usually trade "TBA plus {x} ticks" or a "pay-up" depending on characteristics. These are called "specified pools" since the buyer specifies the pool characteristic he/she is willing to "pay up" for.

The Data

-- current market rate for 30-year FNMA-conforming residential mortgages

-- 1 month TBA prices for agency pass-through securities (FNMA pools)

-- period of study: May 2003 to Nov 2009

TBA: ``Placeholder'' or forward contract which forecasts the price at which pools will trade. Similar to a T-bond futures contract.

A short TBA will deliver a pool, or MBS, with certain characteristics (coupon, WAC, maturity, etc)

A long TBA position takes delivery of the MBS on expiration date.

Analysis

1. For each date in the sample, record the current mortgage rate (R).

2. Calculate 1-day returns for 1-month TBAs for all available liquid coupons

3. Associate a moneyness to each TBA (Coupon-Current Mortgage Rate)

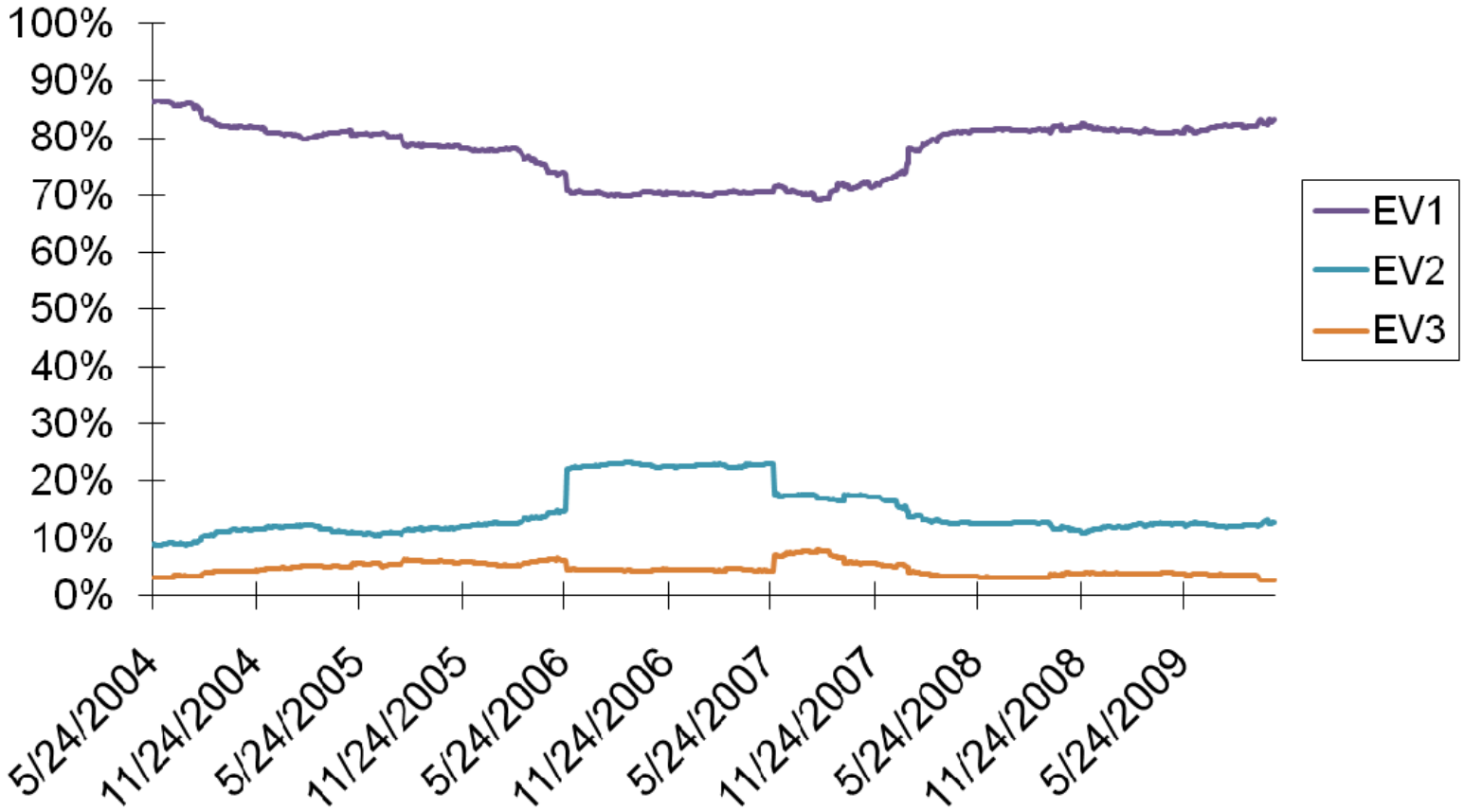
4. Consider the panel (matrix) data consisting of daily TBA price returns, interpolated and centered around the current mortgage rate

>> Analogy with option pricing in terms of moneyness (as opposed to strike price)

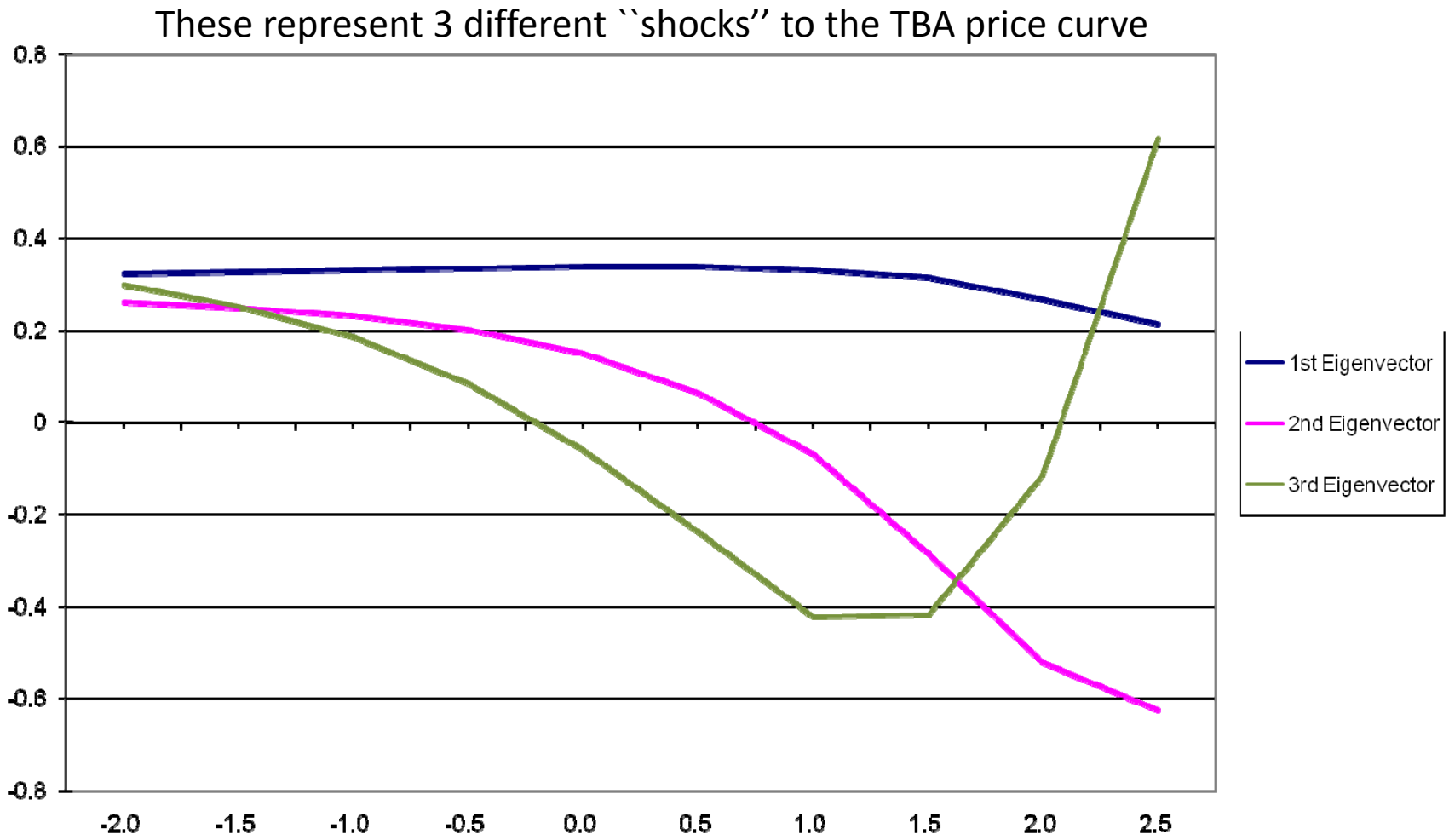
5. Perform PCA analysis and extreme-value analysis for the corresponding factors

>> I-year (252 days) rolling window, ~ 10 liquid TBAs

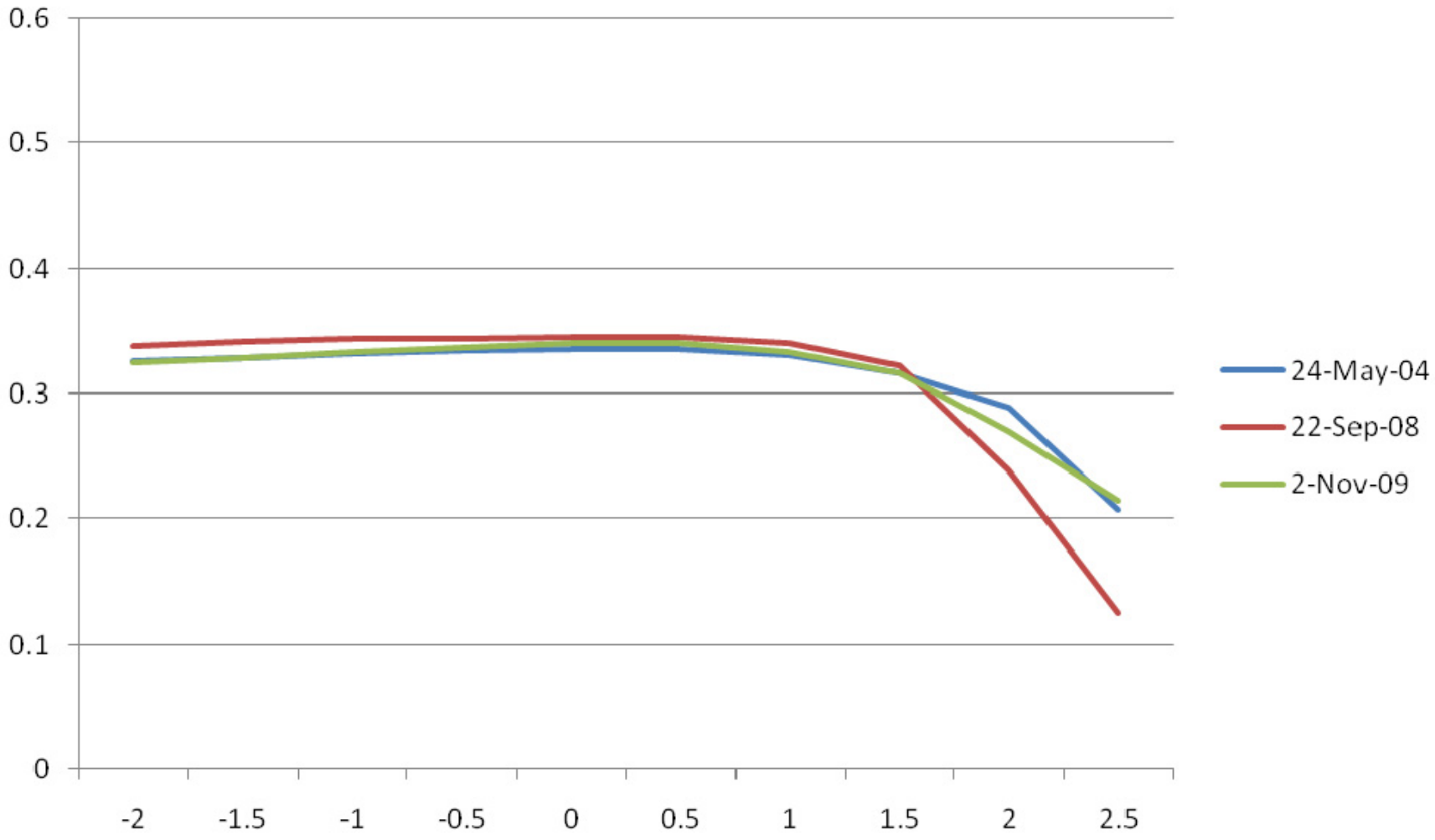
Evolution of 3 largest eigenvalues in the spectrum of 1-month TBA correlation matrix (2004-2010)



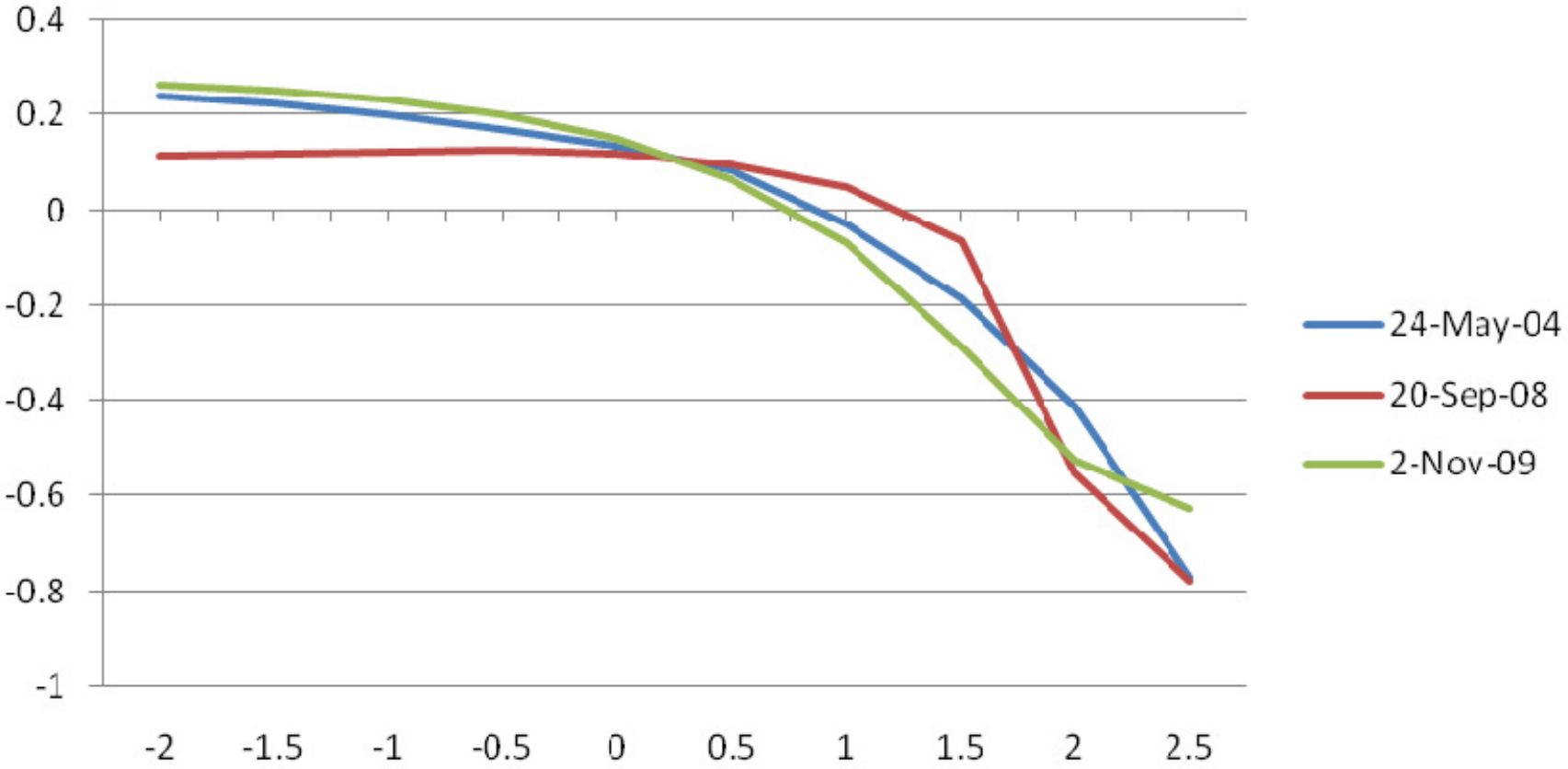
Typical Shapes of the top 3 eigenvectors (taken on 11/2/2009



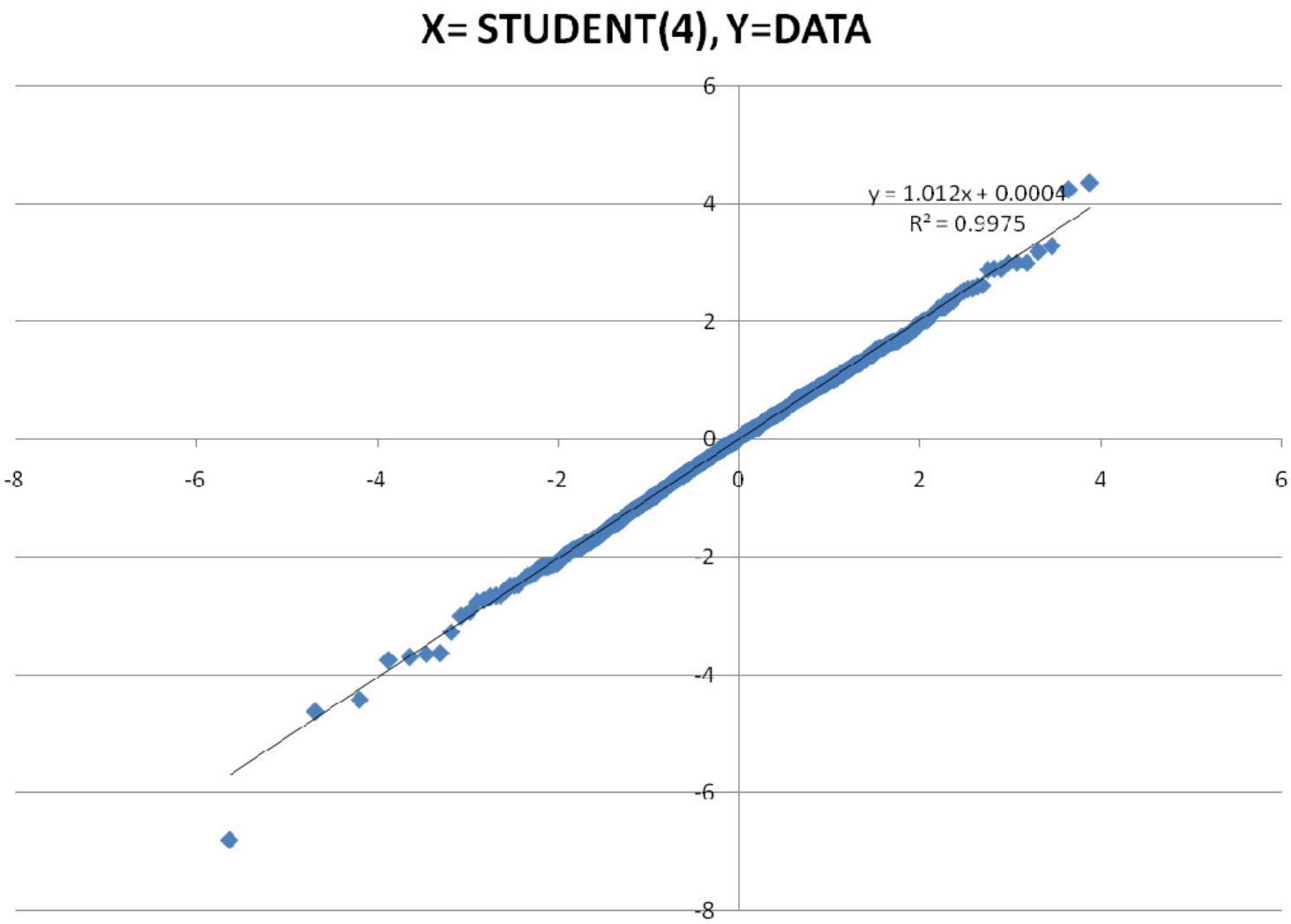
Stability of the first eigenvector



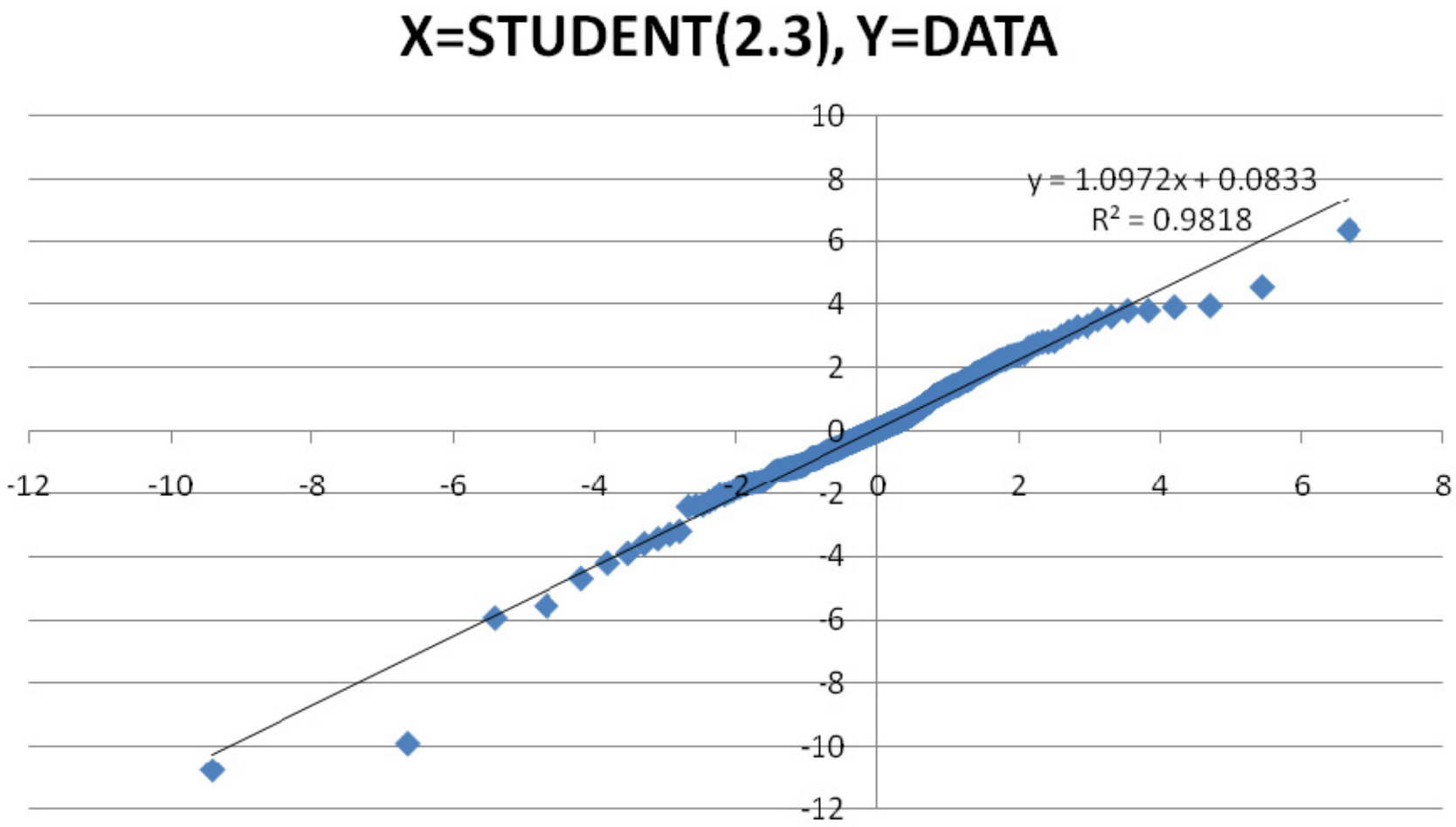
Stability of the Second Eigenvector



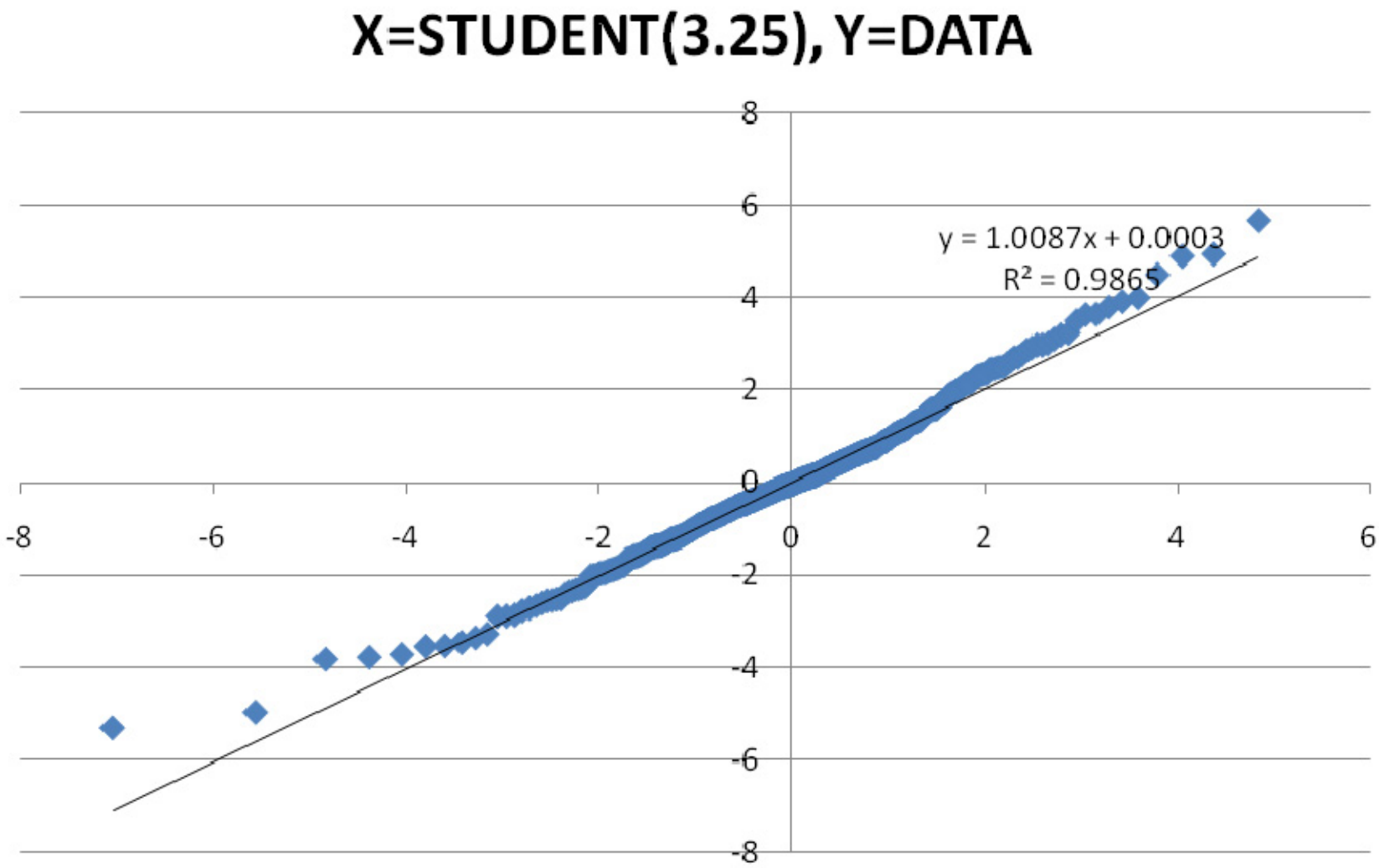
Extreme-value analysis for the tail distribution of the first factor vs. Student(4)



Extreme-value analysis for the tail distribution of the second factor vs. Student(2.3)



Extreme-value analysis for the tail distribution of the third factor vs. Student(3.25)



Statistical Prepayment Modeling

Look at pool data

Organize by moneyness= WAC- (current mortgage rate)

Compute the returns for all pools in the same bucket

-- prepayment (Face Value drop) once a month

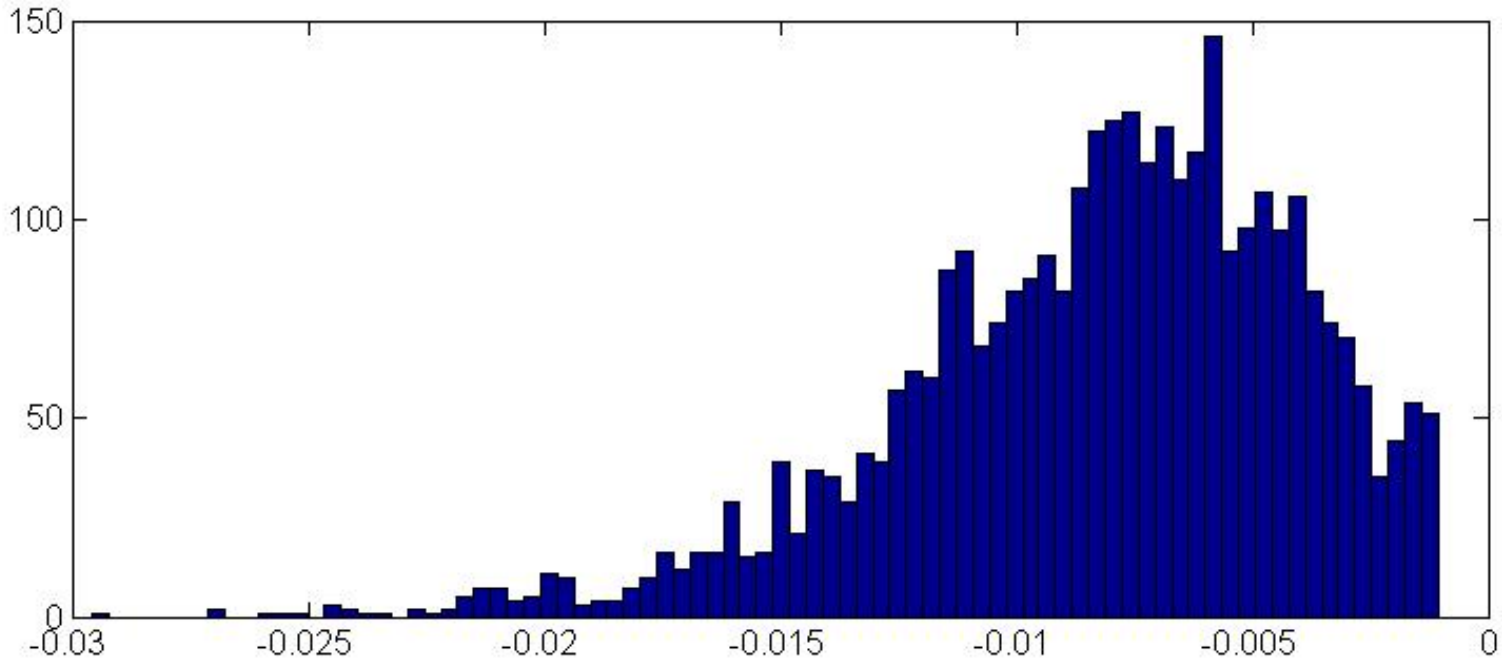
-- TBA variation, every day

Bucketing FNMA returns according to moneyness

C= WAC
R= current mortgage rate

Bucket	Moneyness (C-R)	
	Lower bound	Upper bound
-2	-	-1.75
-1.5	-1.75	-1.25
-1	-1.25	-0.75
-0.5	-0.75	-0.25
0	-0.25	0.25
0.5	0.25	0.75
1	0.75	1.25
1.5	1.25	1.75
2	1.75	2.25
2.5	2.25	-

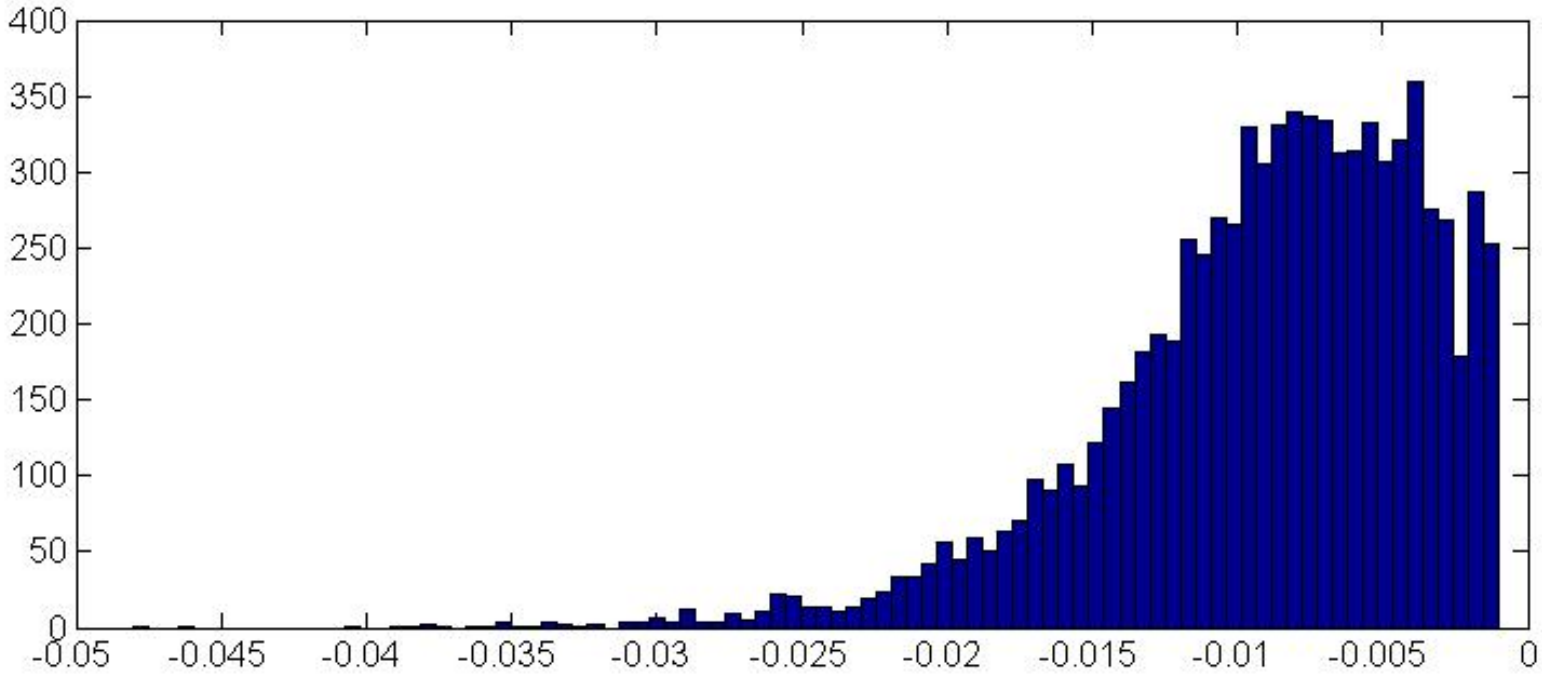
Histogram of monthly prepayments: WAC-Rate=-0.5 (``discount'' bond) (~8000 data points)



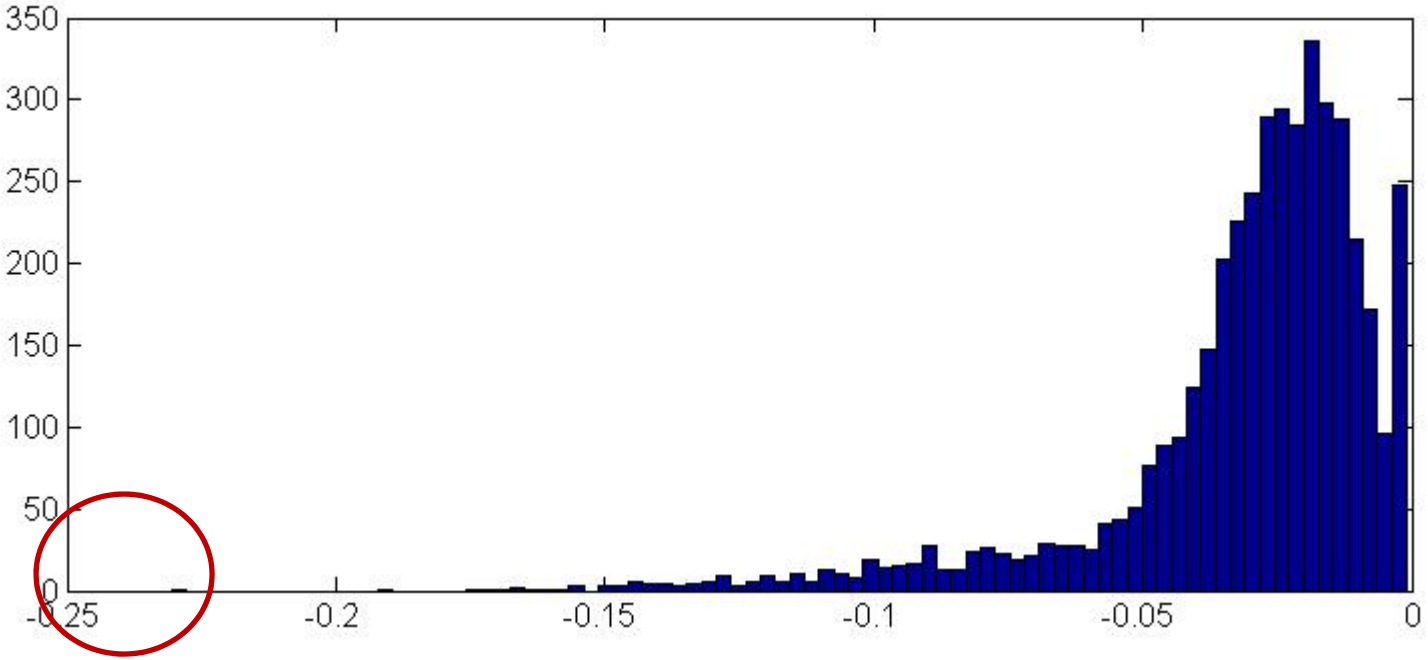
Discount bond= price < 100

Holders of discount bond prefer fast prepayment

Histogram of monthly prepayments: WAC-Rate~0 (``par'' bond)



Histogram of monthly prepayments: WAC-Rate=+0.5 (``premium'' bonds)

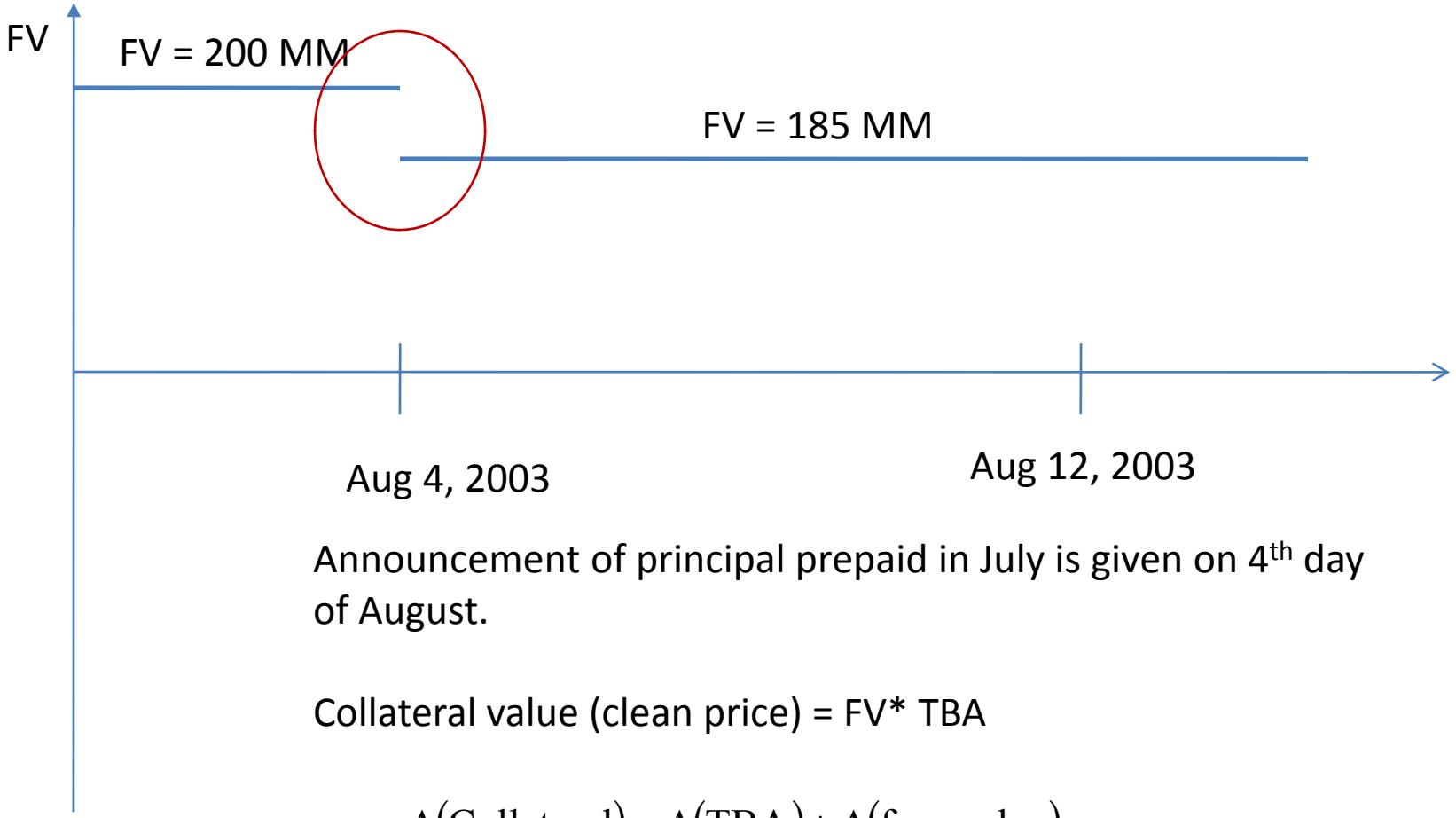


Premium bond= price > 100

Holders of premium bonds prefer slow prepayment

Premium bonds present the largest prepayment risk & extreme values

Application to collateral risk-management

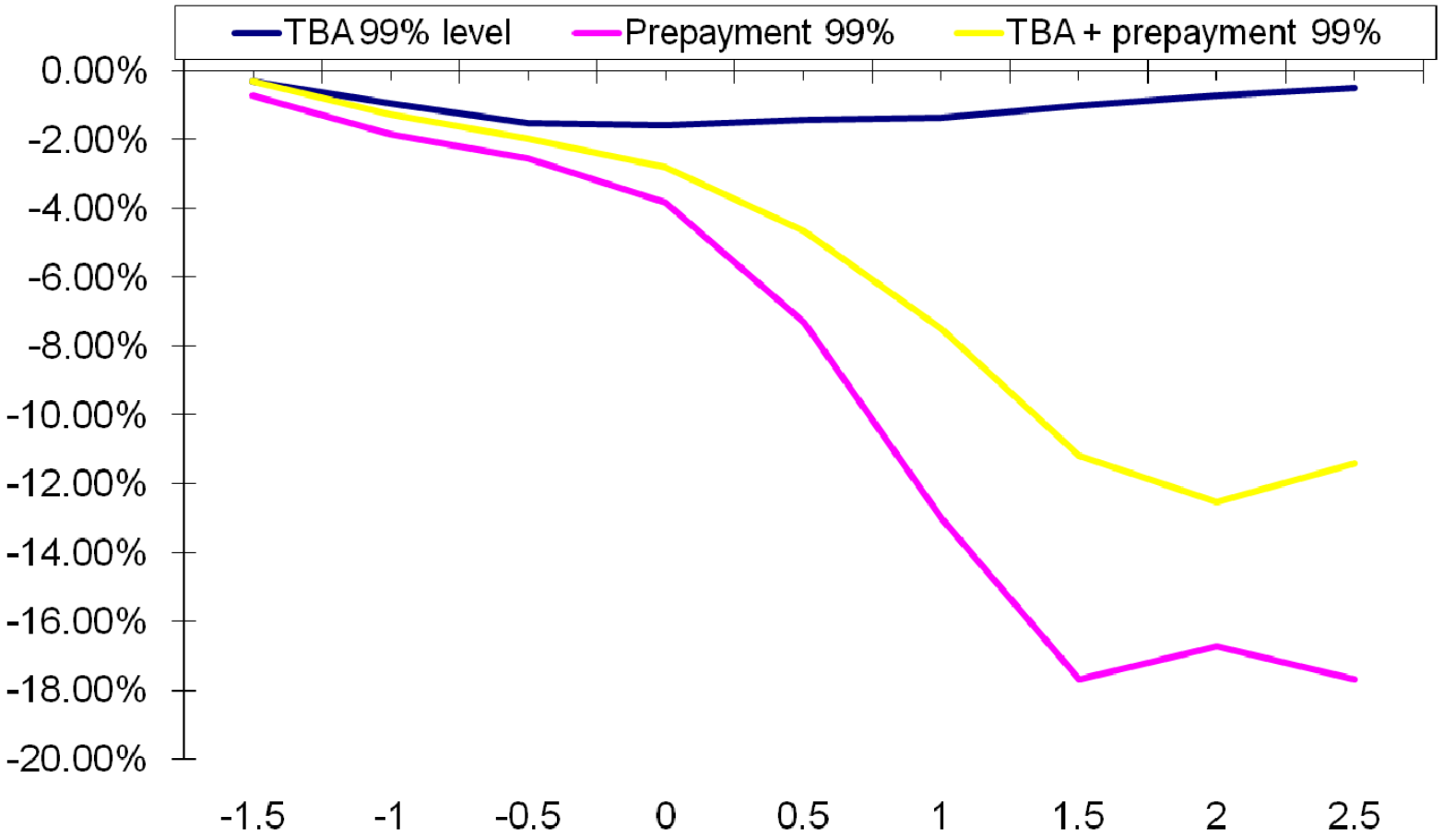


Announcement of principal prepaid in July is given on 4th day of August.

Collateral value (clean price) = FV* TBA

$$\Delta(\text{Collateral}) = \Delta(\text{TBA}) + \Delta(\text{face value})$$

99 % loss levels for MBS pools grouped by moneyness



99 % levels for TBA and Face Value Variations in MBS pools

moneyness	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
TBA 99% quantile	-0.28%	-0.94%	-1.50%	-1.58%	-1.41%	-1.35%	-0.98%	-0.69%	-0.46%
FV 99% quantile	-0.71%	-1.84%	-2.53%	-3.81%	-7.31%	-13.01%	-17.69%	-16.73%	-17.70%
combined 99% quantile	-0.31%	-1.28%	-1.98%	-2.81%	-4.68%	-7.51%	-11.19%	-12.54%	-11.41%

These considerations can be useful to measure exposure on collateralized loans

Notice that the combined quantile is less because of much less instances of changes in FV reported (1/month)

Tails of FV drop can be fitted to power-laws, corresponding to Student with DF~4

Dollar Roll

Dollar rolls are trades designed to deliver a specific security into a TBA contract.

Traders perform dollar rolls to hedge MBS inventory or to take advantage of specific aspects of a pool that would make it attractive to deliver into the TBA.

1. Buy a conforming MBS pool, settlement (T_0)
2. Short a TBS for next-month delivery (T_1)
3. Repo the MBS from T_0 to T_1

Schematic Accounting for Dollar Roll PNL

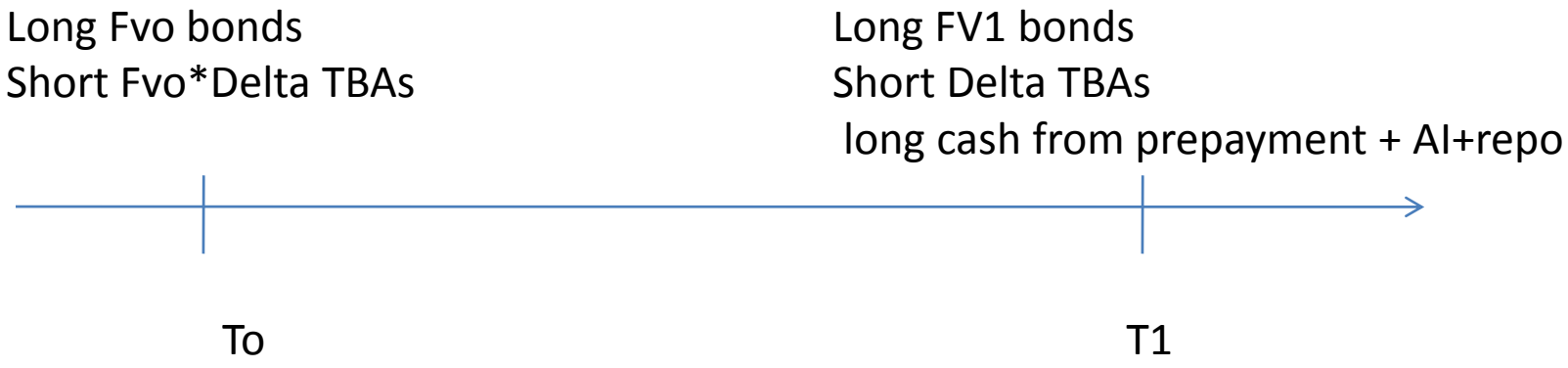
F_0 = face value at inception

F_1 = face value at termination date (after payment of principal)

P_0 = TBA at inception

P_1 = TBA at termination date

Δ = Delta, or hedge ratio of bonds purchased against 1 TBA shorted



Delta is chosen to get zero PNL under TBA SMM forecast

- Bond MTM $= F_1 P_1 - F_0 P_0$

- Prepayment + Interest $= (F_0 - F_1) \cdot 100 + AI \cdot F_0$

- TBA MTM $= -\Delta F_0 (P_1 - P_0)$ $\triangle$ = 1-SMM-SP

If $F_1 = (1 - SMM_0 - SP) F_0, \quad \sigma = SMM_0 + SP$

$\text{MTM} = (1-\sigma) F_0 P_1 - F_0 P_0 + (F_0 - (1-\sigma) F_0) \cdot 100 + AI \cdot F_0 - \Delta F_0 (P_1 - P_0)$

$\text{MTM} = 0 \quad \Rightarrow$

$\Delta = 1 - \sigma \quad \therefore$

$-F_0 P_0 + \sigma F_0 100 + AI \cdot F_0 + (1-\sigma) F_0 P_0 = 0$

$\sigma F_0 (100 - P_0) + AI \cdot F_0 = 0, \quad \sigma (100 - P_0) + AI = 0$

$$SMM_0 + SP = \frac{AI}{P_0 - 100}$$

Final Formula

$$\text{Dollar Roll PNL} = \left(F_1 - (1 - SMM - SP)F_0\right)\left(P_1 - 100\right)$$

Difference between forecast FV and actual final FV

Cost of buying the difference to deliver into a TBA

Statistics for pools with more than 100 MM dollars in face value

DOLLAR ROLL SIMULATION PNL	
Mean	-2E-16
Standard Error	0.00608
Median	0.00755
Mode	-1.4896
Standard Deviation	1
Sample Variance	1
Kurtosis	15.8066
Skewness	-1.4649
Range	17.579
Minimum	-10.814
Maximum	6.76479
Sum	-6E-12
Count	27063
Largest(1)	6.76479
Smallest(20)	-8.2322

	100 MM notional	Basis points
Charge for 10 std	344,742	35
Charge for 20 std	689,484	70

Fitting to Student T (DF=3)
X=Student 3,Y=Data