ICBI Global Derivatives, Amsterdam, 2014

Modeling Volatility Risk in Equity Options: a Cross-sectional approach

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Modeling Volatility Risk

- Equity options depend on the underlying price as well as on the implied volatility of the option
- Volatility returns are correlated to stock returns and to other volatilities
- In the Post Lehmann era, VIX-related products and their options proliferated
- Strategies involving, VIX futures, VIX ETNs and SPX-SPY require proper risk and portfolio management
- Dispersion trading (index options vs. component options/ options vs. options) also requires understanding of correlations between implied volatilities and stocks.
- This presentation gives some recent results on how to think about cross-sectional equity volatility.

Outline

- Classification of equities into ``systemic'' and ''idiosyncratic'' categories based on the fluctuations of their volatility surfaces

- Dimension-reduction and parsimonious descriptions of volatility surfaces

- Cross-sectional analysis of 3800 optionable stocks and their options in a single model

Main tools used : Elementary data analysis, Principal Component Analysis and Random Matrix Theory (Marcenko-Pastur, Tracy-Widom).

I. Classification of equities
based on the fluctuations of their IVS

The Data

- Data source: IVY OptionMetrics (available at WRDS), which gives EOD prices from OPRA

- Data format: Snapshot of Implied Volatility Surface (IVS) parameterized in terms of delta and time-to-maturity (constant delta, constant maturity)

- Size of the problem: 7000 optionable securities with 130 delta-maturity points for each security: approximately 910,000 variables

- This study: 3800 optionable securities with 52 (call) delta-maturity points per underlying asset + underlying asset

$$\delta = (20,25,30, \ldots, 75,80,100), \quad \tau=(30,91,182,365)$$

- Historical period: August 31, 2004 to August 31, 2013

The statistical Analysis

- For each underlying stock, ETF or index, we form the matrix

$$X = \begin{bmatrix} X_{1,1} & \cdots & X_{1,53} \\ \vdots & \ddots & \vdots \\ X_{T,1} & \cdots & X_{T,53} \end{bmatrix}$$

T=1257

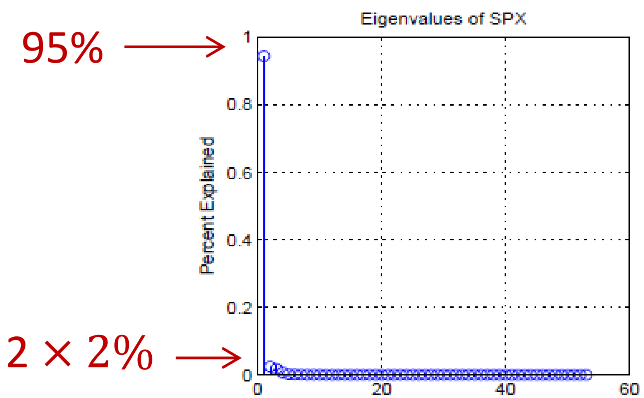
$X_{t,i}$ = standardized returns of stock (i=1) or IVS point labeled i

- Perform an SVD of the volatility surface for each underlying asset in the dataset.

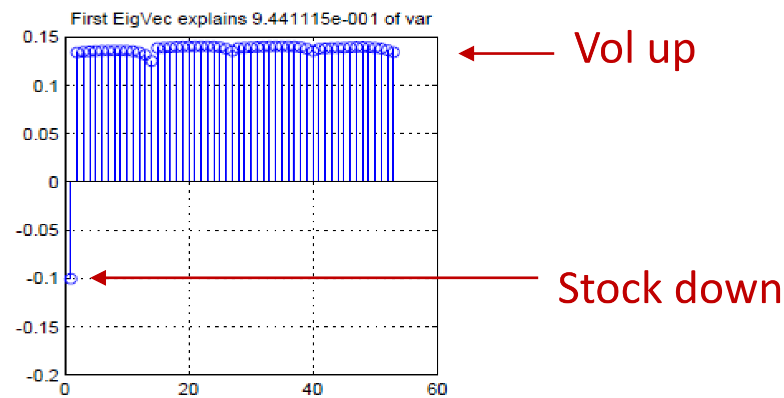
- Analyze eigenvectors and eigenvalues

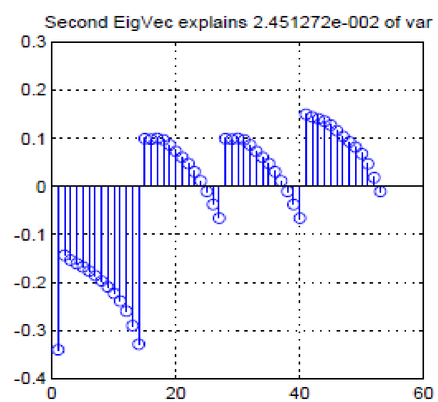
Analysis of SPX volatility surface

Spectrum

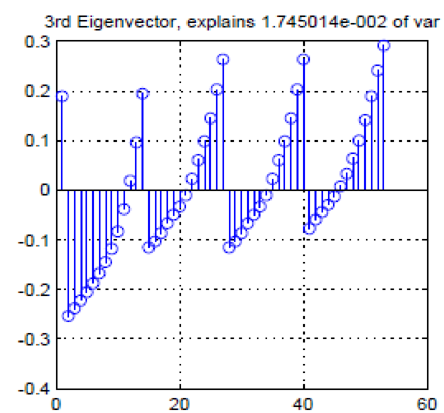


First eigenvector



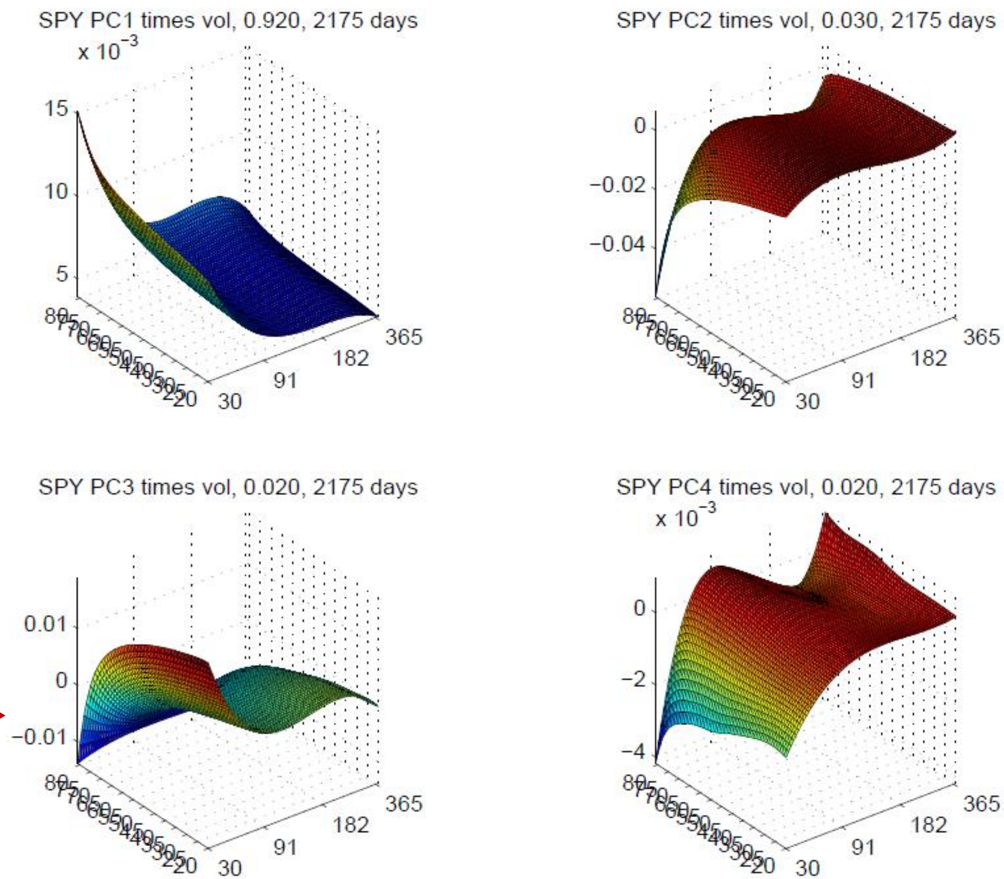


Second eigenvector



Third eigenvector

Main Principal Components for IVS of SPX options

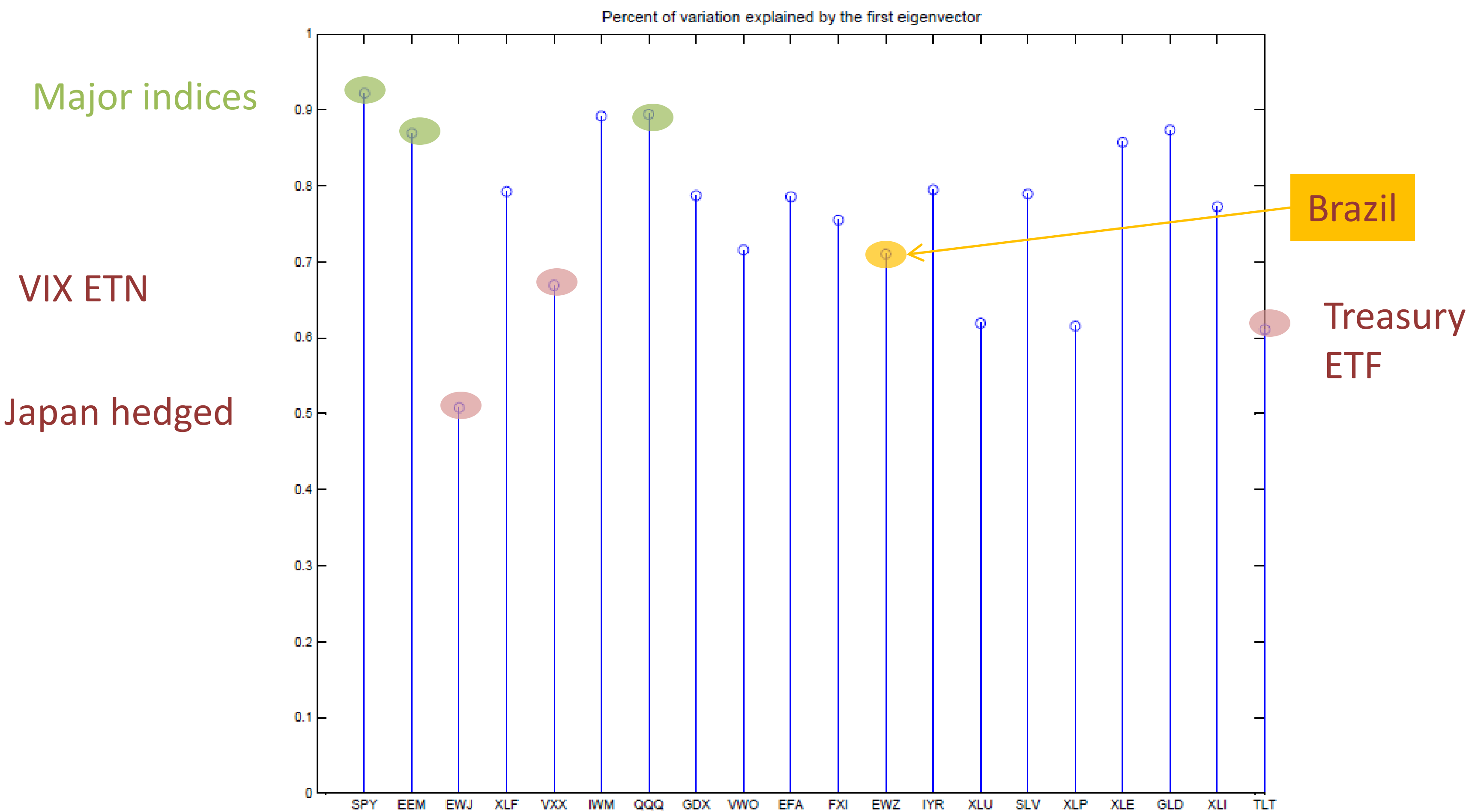


Time-delta movements are coupled

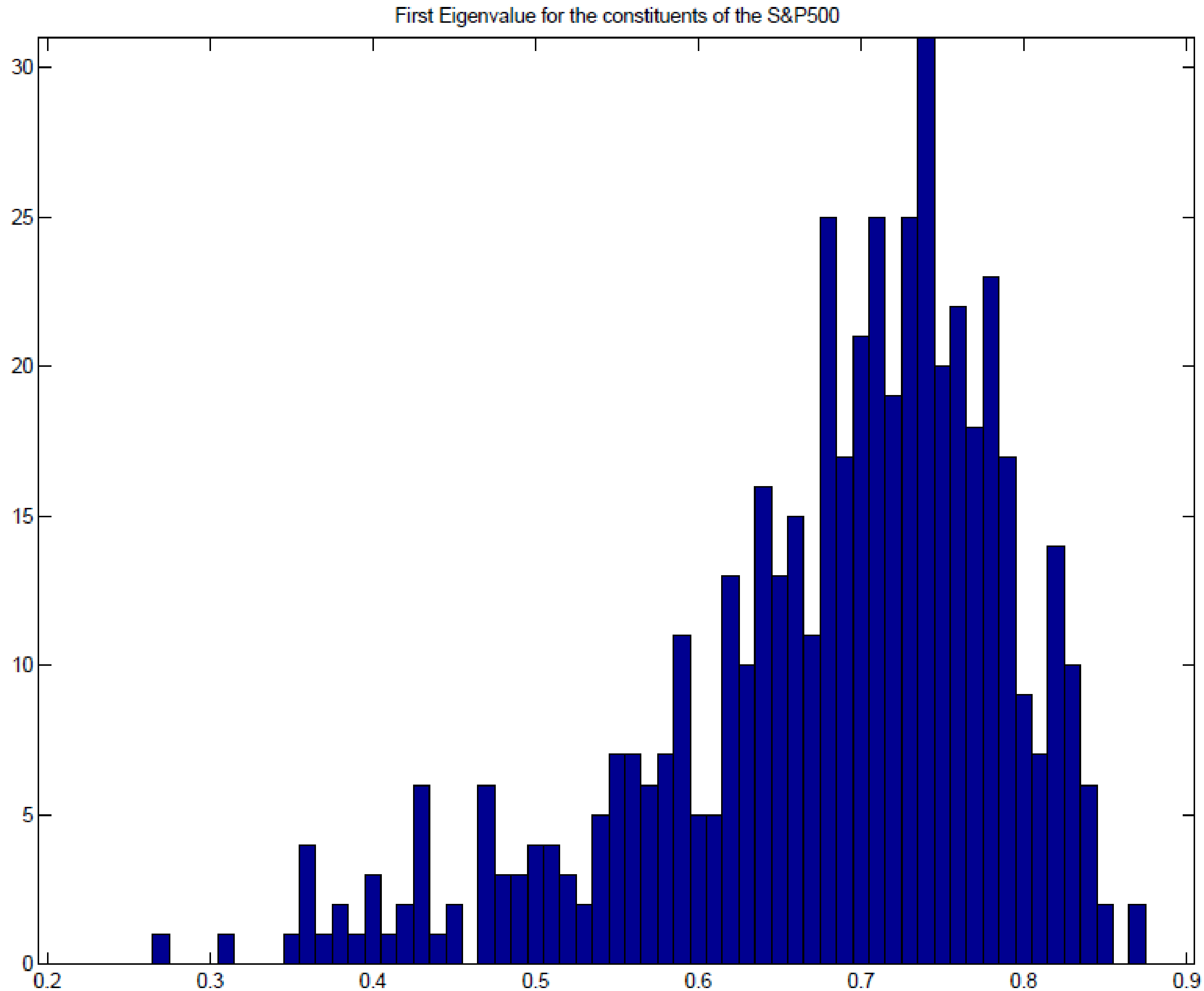
First four principal components of the implied volatility surface for SPX.

20 most liquid ETFs

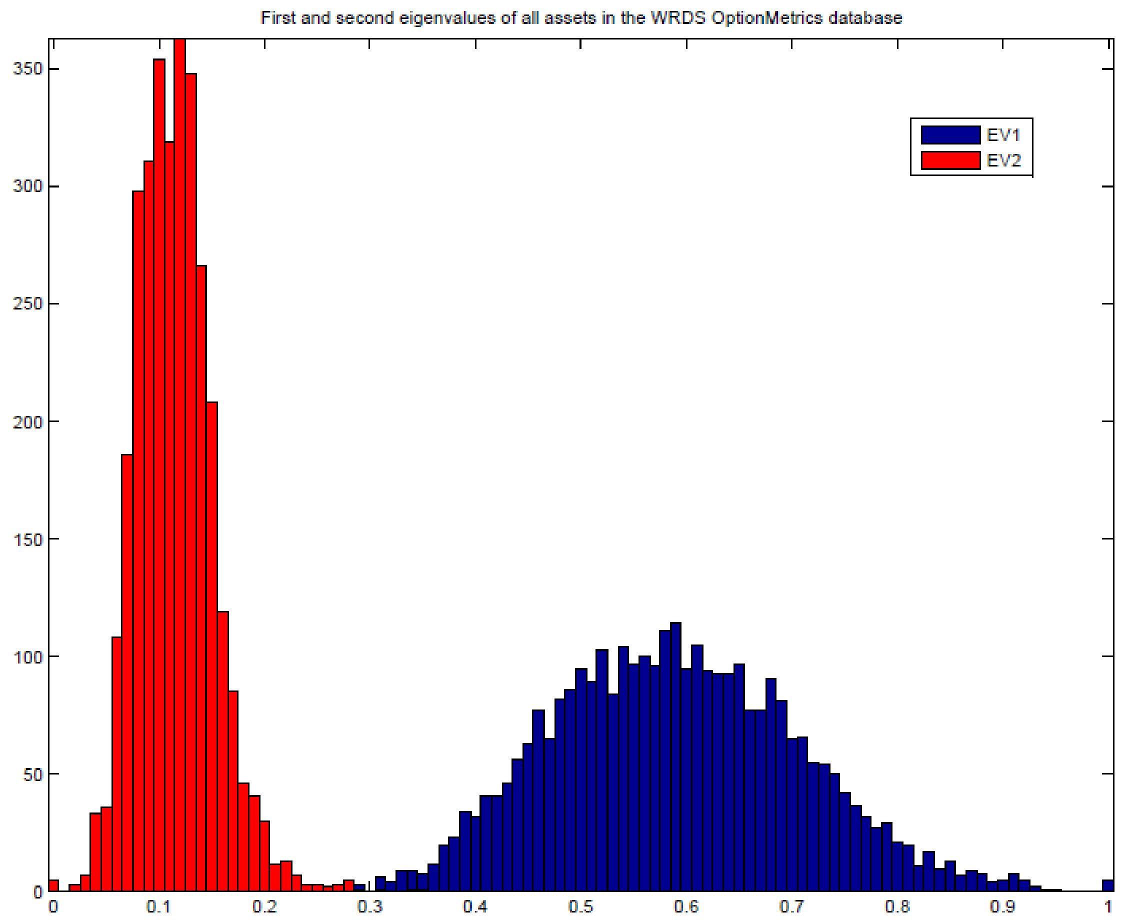
The degree to which the 1st EV explains fluctuations varies from asset to asset



Histogram of first EV of IVS for all constituent stocks of S&P 500



Histogram of 1st and 2nd EVS for all equities in the study



Top and bottom stocks ranked by EV

Table 1.2: Top 15 underlying constituents and bottom 15 underlying constituents by first eigenvalue.

Bottom 15 Constituents	EV1	Top 15 Constituents	EV1
KMI	0.27272	GS	0.87176
POM	0.31406	JPM	0.87092
WEC	0.34958	BAC	0.85115
PNW	0.35611	SLB	0.84669
HCBK	0.35995	CAT	0.84219
NLSN	0.36338	AAPL	0.84126
TE	0.36342	XOM	0.84109
NU	0.3667	NOV	0.83737

Table 1.3: Remaining top 15 underlying constituents and bottom 15 underlying constituents by first eigenvalue.

Bottom 15 Constituents	EV1	Top 15 Constituents	EV1
BMS	0.37675	CME	0.83639
XEL	0.3828	MA	0.8358
WIN	0.38664	MS	0.83471
RSG	0.39779	APA	0.83081
FTR	0.40043	GOOG	0.83011
MKC	0.40475	HIG	0.8301
XYL	0.40596	HES	0.82965

Classification: we can view equities as ``systemic'' or ``idiosyncratic''

- Systemic equities, by definition, have large EV1 (in % terms)
- Idiosyncratic equities have low EV1. In general they have higher EV2, EV3,…
- Idiosyncratic equities are largely affected by corporate events and company specific news. The skew in the IVS (non-parallel IVS shifts) are more important than in systemic stocks.
- Idiosyncratic stocks have typically lower capitalizations and can be subject to take-overs, can have larger earning surprises/ weak earnings guidance, subject to surprises (biotechnology, social media, games), etc.
- Systemic stocks are very much driven by the market risk appetite (risk on, risk-off).

Significance of higher-order EVs Analysis of the IVS spectra using RMT

- An important question – going beyond the first EV – is to find out how many eigenvectors are significant.

- Random matrix theory: if X is a random matrix of IID random variables with mean zero and variance 1, of dimensions $T \times N$, the density of states of the correlation matrix

$$C = \frac{1}{T} X X'$$

approaches a N and T tend to infinity with ratio N/T=γ the Marcenko-Pastur distribution

$$\frac{\#\{\lambda : \lambda \leq x\}}{N} \quad \rightarrow \quad MP(\gamma; x) = \int_0^x f(\gamma; y) dy$$

$$\mathrm{N} \rightarrow \infty, \frac{N}{T} \rightarrow \gamma$$

Marcenko-Pastur threshold

$$f(\gamma; x) = \left(1 - \frac{1}{\gamma}\right)^{+} \delta(x) + \frac{1}{2\pi\gamma} \frac{\sqrt{(x - \lambda_{-})(\lambda_{+} - x)}}{x} \qquad \lambda_{-} \leq x \leq \lambda_{+}$$

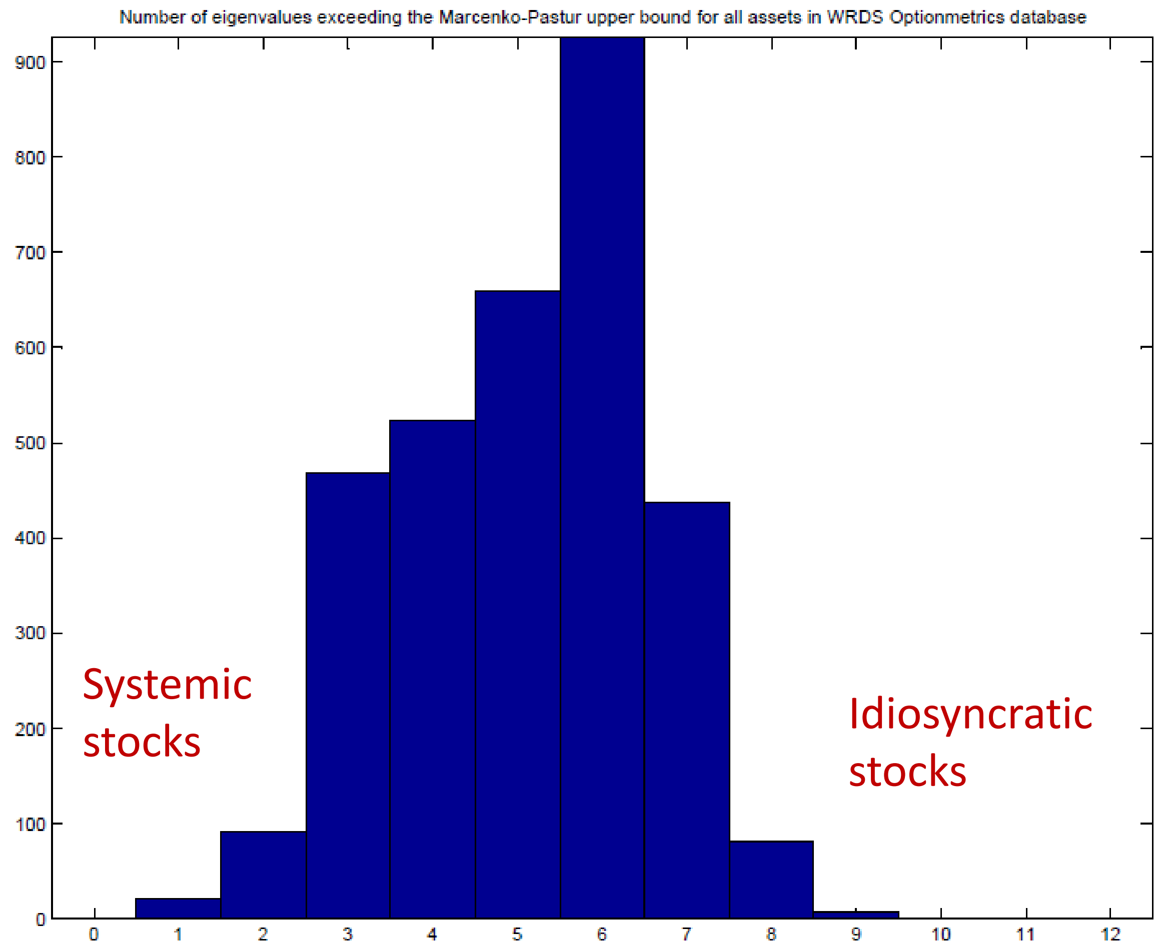
$$\lambda_{-} = (1 - \sqrt{\gamma})^{2} \quad \lambda_{+} = (1 + \sqrt{\gamma})^{+} \longleftarrow$$

Marcenko-Pastur threshold

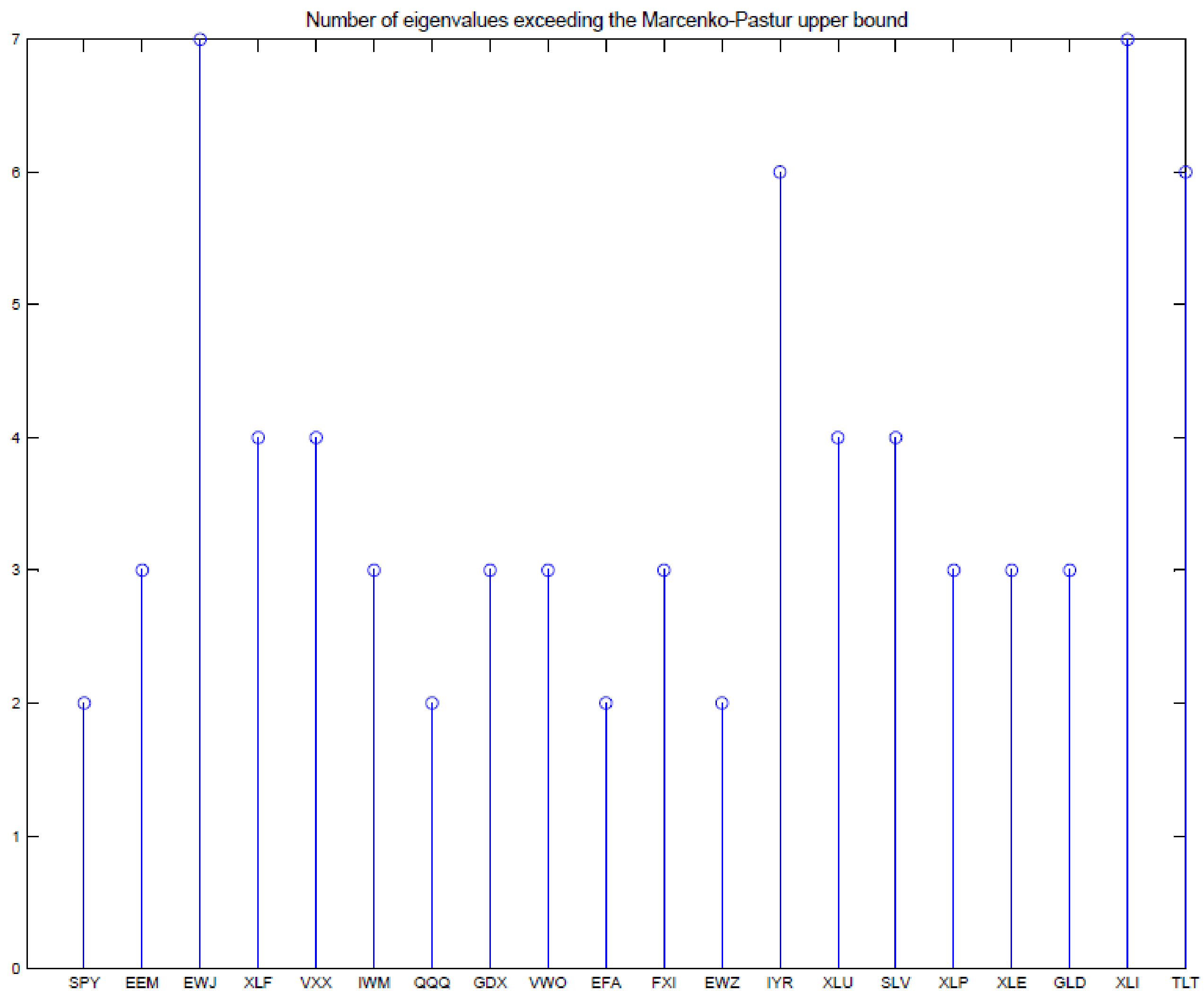
The theoretical top EV for the IVS is $\lambda_{+} = \left(1 + \sqrt{\frac{53}{1257}}\right)^{2} = 1.45$

- Eigenvalues of the correlation matrix which correspond to non-random features should lie above the MP threshold (within error)
- The idea was developed in Laloux, et al (2000) and Bouchaud and Potters (2000) for studying equity correlations

Number of EVs above the MP threshold can be large for idiosyncratic stocks



Systemic stocks correspond to simple dynamics for their IVS



EV1 is negatively correlated to Ev(n) and to the # of significant EV

Cross-sectional correlation matrix of EV1,...,EV4 and #EV>MP

EV1	EV2	EV3	EV4	# EVs > MP
1.00	-0.77	-0.85	-0.94	-0.89
-0.77	1.00	0.72	0.74	0.62
-0.85	0.72	1.00	0.81	0.72
-0.94	0.74	0.81	1.00	0.88
-0.89	0.62	0.72	0.88	1.00

This confirms that the dynamics of IVS for systemic stocks (EV1>0.8) are simpler than for idiosyncratic stocks (EV1<0.4).

Traders and risk managers should be aware of this.

II. Dimension reduction and parsimonious descriptions of IVS

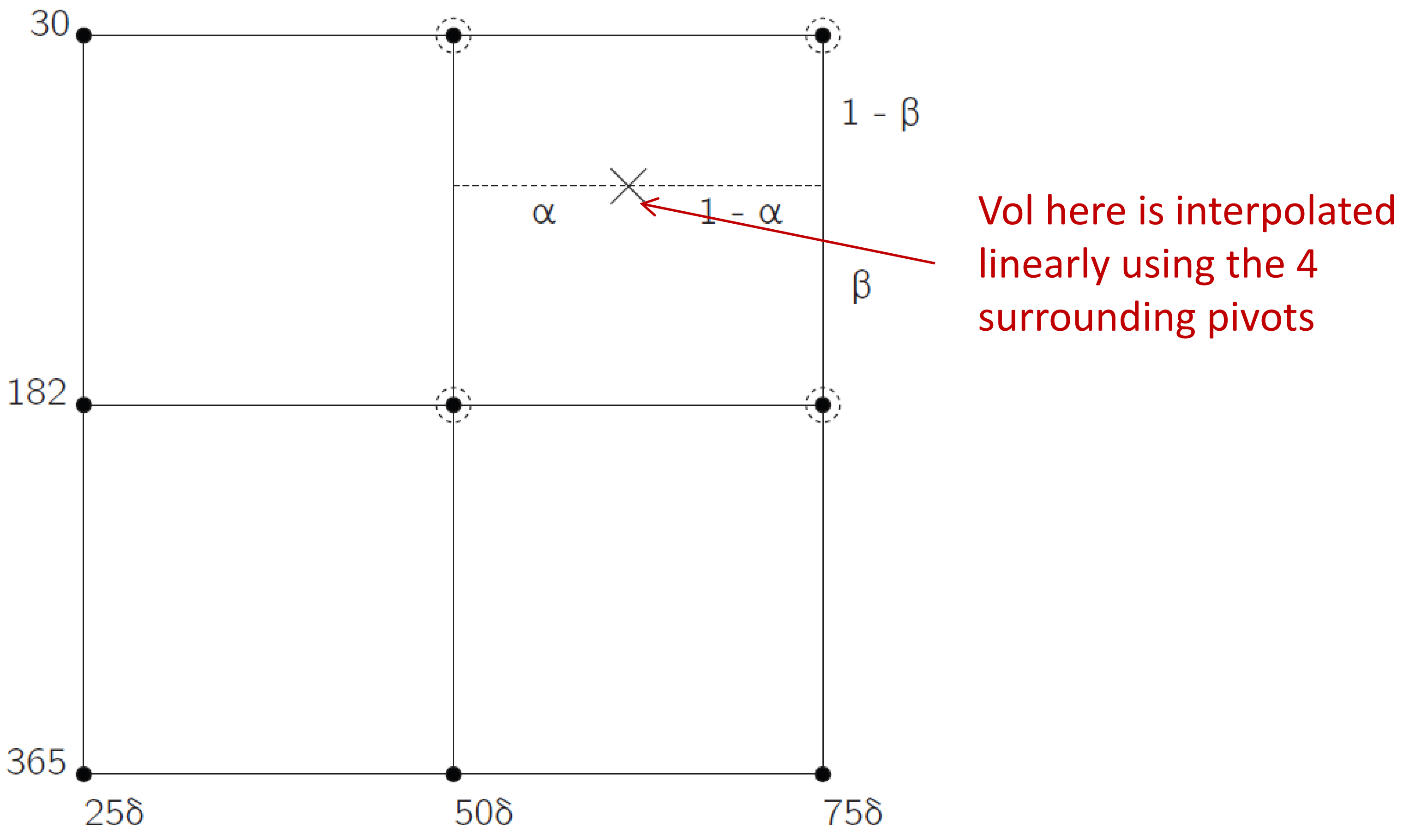
Dimension reduction

- We have seen that IVS move rather simply for systemic names
- Dynamics can be more complicated for idiosyncratic assets
- What is a reasonable number of risk-factors needed to parameterize all IVS?
- We shall use an approach based on picking distinguished points on the IVS (a subset of the 52 or the 130 points given in Option Metrics)
- Pivot: a point on the delta/tenor surface used as a risk factor
- Pivot scheme: a grid of pivots, which will be used to interpolate the remaining implied volatility returns.
- GOAL: find a pivot scheme that approximates well the significant spectrum and EV1 in particular (same grid for all assets!)

The pivot schemes that we tested

	2-Pivots	4-Pivots	5-Pivots	6-Pivots	7-Pivot	9-Pivots	12-Pivots
25δ 30				YES	YES	YES	YES
50δ 30	YES	YES	YES			YES	YES
75δ 30				YES	YES	YES	YES
25δ 91				YES	YES		YES
50δ 91					YES		YES
75δ 91				YES	YES		YES
25δ 182		YES	YES			YES	YES
50δ 182			YES			YES	YES
75δ 182		YES	YES			YES	YES
25δ 365				YES	YES	YES	YES
50δ 365	YES	YES	YES			YES	YES
75δ 365				YES	YES	YES	YES

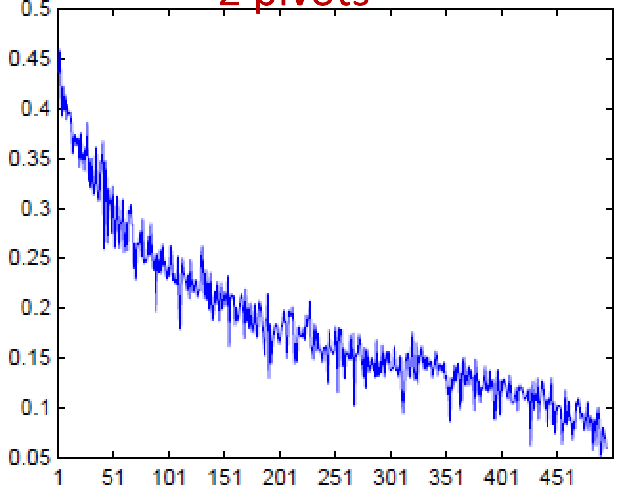
9-pivot scheme



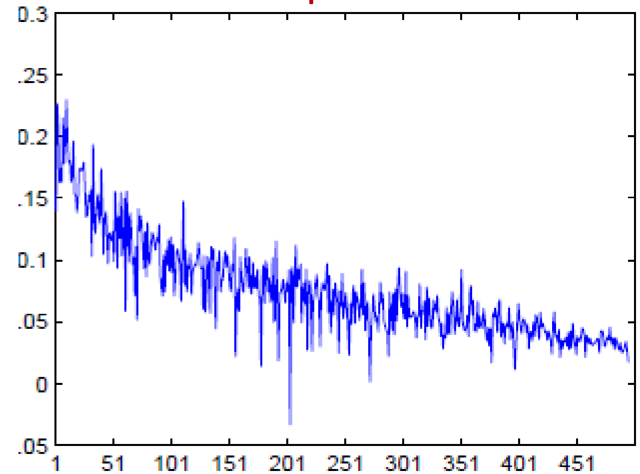
Increasing the number of pivots results in a better approximation of EV1

2 pivots

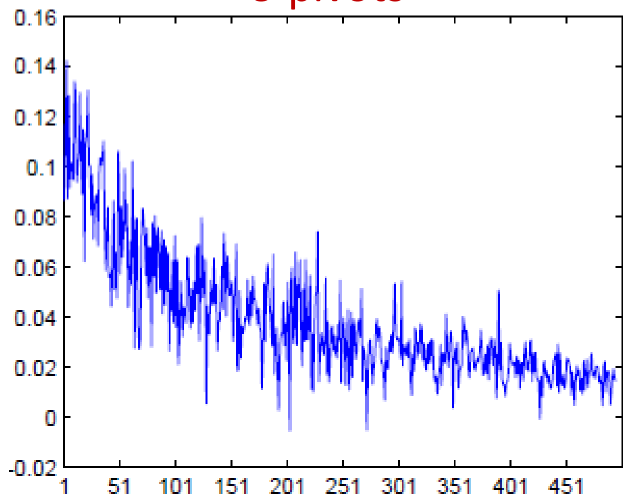
% Error



6 pivots



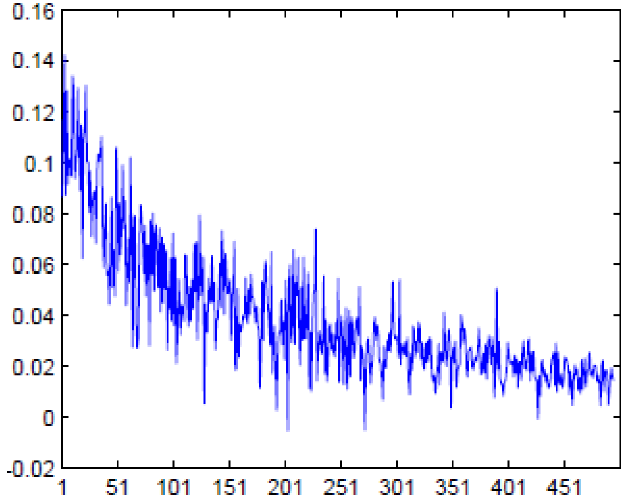
9 pivots



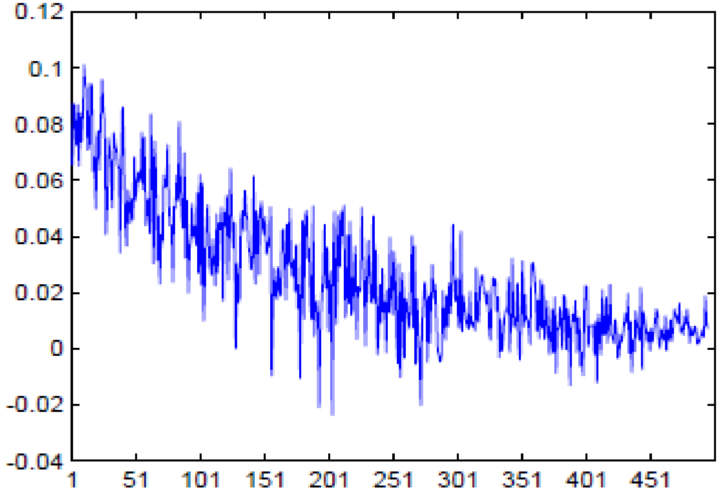
Cross section of S&P 500 constituents.

12- pivot scheme does slightly better, but not much

9 pivots



12 pivots



- 9 pivots seems like an appropriate number to parameterize all the IVS in the data.
- This was confirmed by dynamic PCA with small window (Dobi’s thesis, 2014)

III. Joint correlation analysis for all optionable stocks and their volatility surfaces

The large data matrix

- We determined that for equities and their listed options, the 9-pivot model for each IVS might be sufficient to describe the option market
- We study 3141 equities over 500 days. The dimensionality in column space (number of correlated variables) is N= $3141 \times 10 = 31410$.
  The number of rows is 500.
- We have to model a correlation matrix of 31K $\times$ 31K. This is better than 310,000 by 310,000.
- IDEA: Following Laloux et al, and Bouchaud and Potters; extend their work on equities using Marcenko-Pastur to equities + options.

Marcenko-Pastur Threshold and Main Questions

- The MP Threshold is

$$\lambda_+ \approx \left(1 + \sqrt{\frac{31410}{500}}\right)^2 = 79.67$$

- This suggests that we keep eigenvalues above 79.67 and declare that the rest is noise….

- Question 1: how many EVs exceed (significantly) the 79.67?

- Question 2: Is MP valid for stocks/options given the heavy nature of distributional tails?

- Bouchaud et al have shown that MP does not hold for random matrices in which the coefficients have heavy tails.

Checking that the MP criterion applies

- Take the return matrix X and randomize the order of each column to produce a new matrix Y. The new matrix has same distributions for the entries but columns are uncorrelated

- Do a sample of 10,000 such matrices

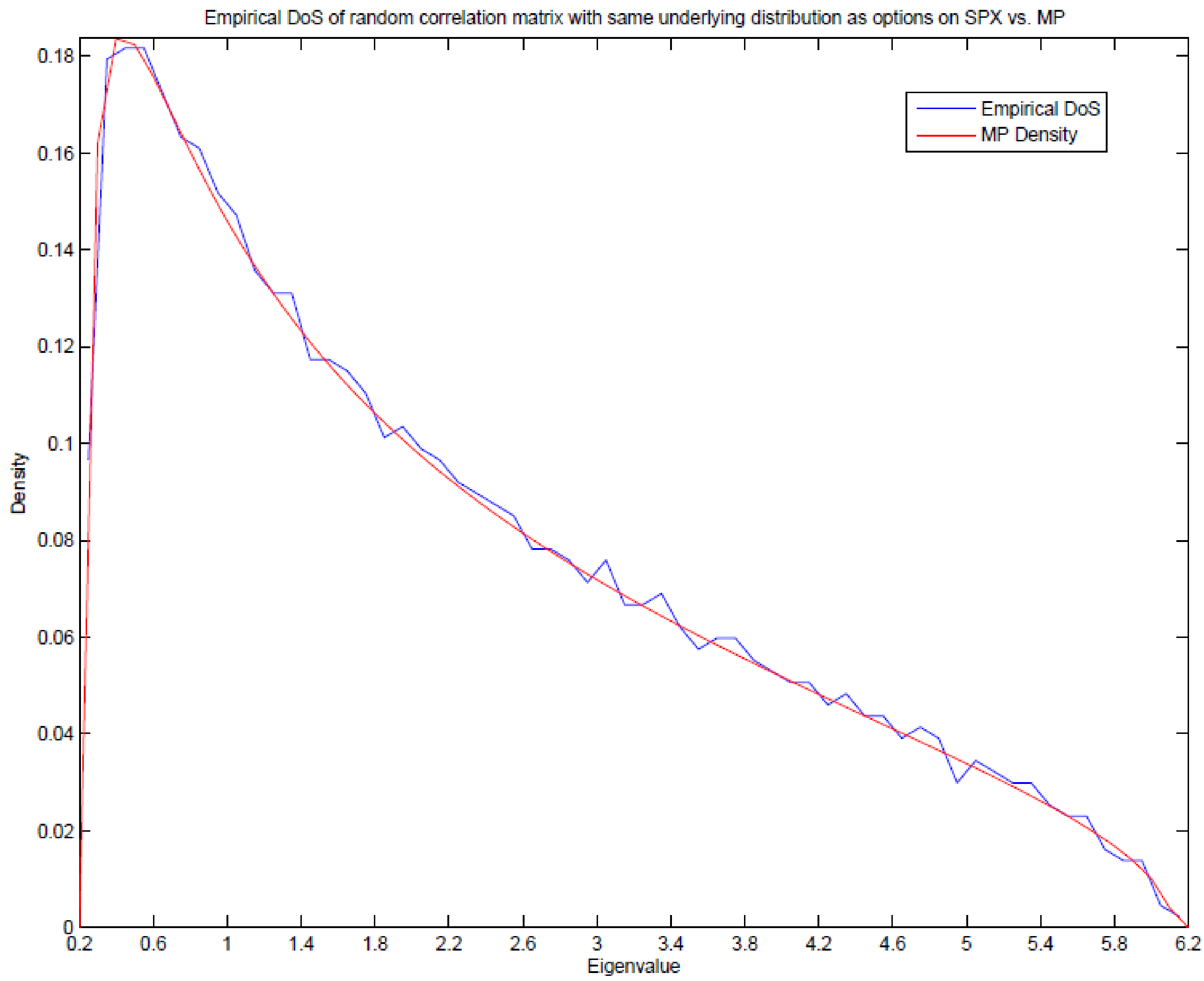
- Compute the average DOS

- Compute the distribution of largest eigenvalue across the sample

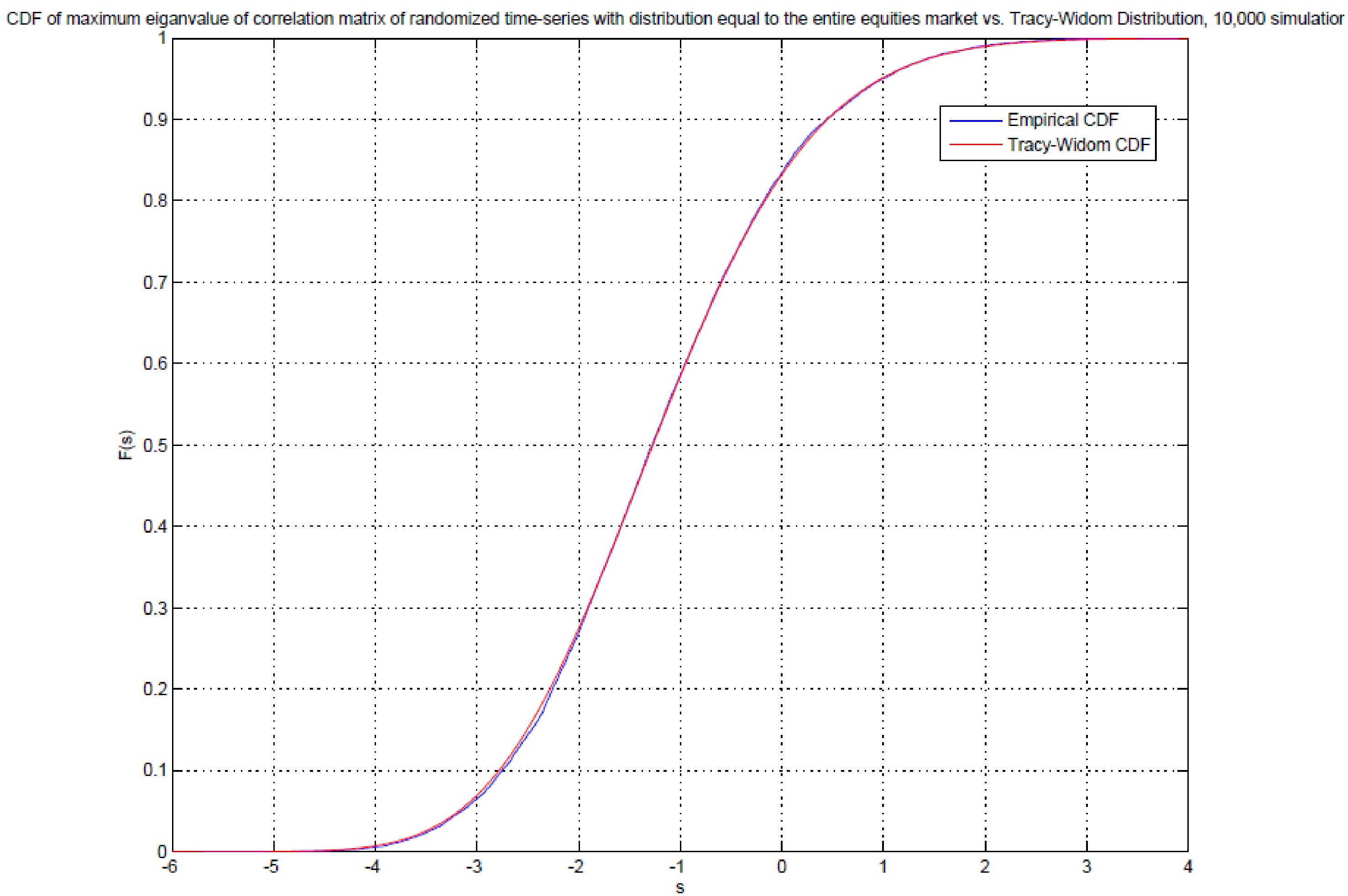
- We did this for 4 ensembles

    1. Constituents of S&P 500 (no options)
    2. All optionable equities for which there was data
    3. S&P 500 with options
    4. All optionable equities with options

Density of States: Empirical vs MP



CDF for maximum eigenvalue: random matrix vs. Tracy-Widom



Main new result:
There are 108 significant Evs in the options market

Top 110 Eigenvalues	s-value	$F_1(s)$
$\lambda_1 = 3742$	24843	1
$\lambda_5 = 209.27$	879.14	1
$\lambda_{10} = 143.5$	433.04	1
$\lambda_{20} = 118.19$	261.32	1
$\lambda_{40} = 102.62$	155.74	1
$\lambda_{50} = 97.40$	120.35	1
$\lambda_{70} = 90.48$	73.35	1
$\lambda_{90} = 84.56$	33.21	1
$\lambda_{107} = 80.21$	3.70	.9996
$\lambda_{108} = 80.04$	2.60	.996
$\lambda_{109} = 79.65$	-.10	.80
$\lambda_{110} = 79.41$	-1.71	.35

MP threshold

Final correlation results

1. Constituents of S&P 500 (no options):
   15 significant eigenvalues, explaining 55% of variance

2. ~3100 stocks from OptionMetrics (no options):
   20 significant eigenvalues, explaining 24% of the variance

3. Constituents of S&P 500 AND options with 9 pivots:
   84 significant eigenvalues, explaining 55% of variance

4. Large dataset + options with 9 pivots:
   108 significant eigenvalues, explaining 50% variance

Conclusions

- We presented an approach to model the statistical fluctuations of the entire US listed derivatives market
- We notice that implied volatility surfaces can have different degrees of shape variability depending on the size of EV1 for different assets.
- We interpret EV1 as a measure of how ``systemic'' a stock is.
- We propose modeling each IVS with 9 pivots or risk factors and claim that linear interpolation using these factors should produce very similar fluctuations as the entire surfaces, across all equities.
- We use this approach to analyze the correlation matrix of a large cross-section of equities and their implied volatility surfaces.
- We find that the number of significant EVs for the U.S. equity derivatives market is approximately 108.
- For more information please contact Doris Dobi at NYU (doris.dobi@gmail.com) or myself.