New Techniques for Pricing VIX Futures and VXX Options

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Summary

• Commodity Futures and ETFs
• The Contango/Backwardation Effects
• VIX Futures and VIX ETFs
• Applications to VXX/VXZ trading, 2009 -2010
• New applications to VXX options pricing and trading, 2010-2012
Theoretical Commodity Futures Pricing

- Cash & carry costs include transportation and storage and the "convenience" of having commodity to be able to sell it in the spot market and replace it later.

\[ q = \text{(convenience yield)} - \text{(transportation)} - \text{(storage costs)} \]
\[ r = \text{term rate} \]

\[ F_{0,T} = S_0 e^{(r-q)T} \]

- The shape of the forward curve depends on the demand vs. availability in storage, transportation costs and convenience yield.
Futures are said to be in **contango** if the futures price increases with the time-to-delivery (futures is higher than spot).

If the futures are in contango, this means that the "convenience yield" is low and the cost of storing and financing make the forward delivery more expensive as time goes by.
Commodity Futures: Backwardation

Futures are said to be in **backwardation** if the futures price decreases with the time-to-delivery (futures is lower than spot)

Associated with a high convenience yield. For example, rental for gold could be expensive, etc.
WTI Futures Term Structure in September 2011 (contango)
Volatility of Contango: WTI Futures, 2006 to 2011

CL4 - CL1

Contango

Backwardation

Contango

1/3/06  1/3/07  1/3/08  1/3/09  1/3/10  1/3/11
Commodity ETFs: Physical vs. Synthetic

- The ‘equitization’ of commodities, currencies and fixed-income has been made possible by the introduction of ETFs, ETP (exchange-traded products), ETCs (exchange-traded commodities), ETNs, etc.

- ETFs which track commodities are structured by holding the physical commodity (e.g. gold, silver) or by implementing a rolling-futures strategy (USO, VXX)

- Most commodities have storing costs, are impossible to store, or are perishable, so the futures approach is almost inevitable

- It may not make sense economically to store industrial commodities (crude oil, copper) for ETF purposes
Known pitfalls in commodity ETFs: Physical vs. Futures

Futures-based ETFs are based on maintaining a position in a constant-maturity futures by rolling from one contract to the next as they reach maturity.

This may result in underperformance with respect to the spot commodity for at least 3 reasons:

-- rolling costs (front-running)

-- position limits in futures exchanges limits AUM growth and produce distortions

-- rolling costs (contango /backwardation)
Crude Oil ETFs

Major oil ETFs, excluding exotic and inverse funds

• **USO**: Most liquid, Daily volume 8.8 million shares, AUM=1.1 B

• **UCO**: Daily volume 4.5 million shares, AUM= 250 million

• **OIL**: 760,000 shares, AUM = 463 million

• **DBO**: 330,000 shares, AUM = 500 million

• **USL**: 74,000 shares, AUM = 169 million

All the above crude oil ETFs are synthetic. Most of them are based on exchange-traded futures
Futures-based ETFs: the rolling conundrum

Mandate:

-- position in one or more contracts, aiming to carry a fixed-maturity

-- replace (roll) contracts as expiration arrives

\[ \frac{dI_t}{I_t} = a(t) \frac{dF_t^{(1)}}{F_t^{(1)}} + (1 - a(t)) \frac{dF_t^{(2)}}{F_t^{(2)}} + r dt \]

\[ I_t = \text{value of the index at date } t \]

\[ F_t^{(i)} = \text{futures with settlement date } T_i \]
Rolling with constant maturity $\tau$

Discrete rolling (USO, UNG)

$$a(t) = \begin{cases} 
1, & \text{if } t + \tau < \frac{1}{2} (T_1 + T_2) \\
0, & \text{if } t + \tau \geq \frac{1}{2} (T_1 + T_2) 
\end{cases}$$

Continuous rolling (VXX, VXZ)

$$a(t) = \frac{T_2 - (t + \tau)}{T_2 - T_1}$$
Contango implies futures drop towards spot

Use simple model for F

\[ F^{(i)}_t = S_t e^{(r_i - d_i)(T_i - t)} \quad \text{contango} \iff r_i - d_i > 0 \]

\[ S_t = \text{spot price} \]

\[ r_i = \text{rate for expiration } T_i \]

\[ d_i = \text{convenience yield - storage cost for mat. } T_i \]

\[ \frac{dF^{(i)}_t}{F^{(i)}_t} = \frac{dS_t}{S_t} - (r_i - d_i)dt, \]

In a low interest rate environment, contango means that convenience yields are negative. \((d_i < 0)\). Contango= net cost-of-carry is positive.
Consequence for futures-based ETFs

\[
\frac{dI_t}{I_t} = a(t) \frac{dF_t^{(1)}}{F_t^{(1)}} + (1 - a(t)) \frac{dF_t^{(2)}}{F_t^{(2)}} + rdt
\]

\[
= \frac{dS_t}{S_t} - [a(t)(r_1 - d_1) + (1 - a(t))(r_2 - d_2)]dt + rdt
\]

\[
= \frac{dS_t}{S_t} + [a(t)d_1 + (1 - a(t))d_2]dt
\]

Negative drift relative to spot price if convenience yield is negative

Convenience yield < 0 = "hyper-contango".
The USO oil ETF

United States Oil ETF (USO) vs Spot WTI Crude Oil

Spot WTI Crude Oil Prices

USO ETF substantially underperformed spot when crude oil futures were in steep contango

(c) Barchart.com
Dbo is based on a basket of futures with different maturities. It is less vulnerable to contango than USO.
Arbitrage strategies for commodity ETFs

-- Short the worst performing ETF (due to contango, for example)

-- Hedge with futures or the best performing etfs

-- take into account the borrowing costs for establishing the short position

-- dynamic rebalancing of positions to limit risk

  -- daily
  -- periodically (with a rule?)
  --``buy and hold’’, risk-managed
VIX: the "commodity" of equity markets

- VIX Futures = Futures contracts with monthly expirations settling on spot VIX. 10 monthly expirations are actively traded with considerable liquidity in the 3 front months.

- VIX is generally in contango (like index option volatility)

- In a trending market, option volatility is higher for longer maturities unless the market is very stressed.

- During a market dislocation, traders "bid up" near term volatility producing Backwardation in the VIX futures

- Volatility is "stored" in SPX options and variance contracts, but is not easily converted to VIX and vice versa. No easy cost-of-carry formula!!
VIX and VIX Futures (March 22, 2011)

VIX is in contango (this is the general form of the VIX futures curve)
Stressed Equities Market: backwardation (demand for short-term protection)
Flash Crash (6 May 2010)
The VXX and VXZ ETNs

VXX: iShares ETN which tracks short term VIX futures (months 1 and 2)
    target maturity 30 days; continuous roll

VXZ: iShares ETN, tracks mid-term VIX futures (months 4 through 7);
    target maturity 120 days; continuous roll

Both securities have negative drift and are correlated to the same underlying asset.

The VIX curve, in natural contango, tends to be **steeper for shorter Maturities** (like most vol curves)

This gives rise to the possibility of arbitrage by building a long-short position
VXX and VXZ ETNs are very popular by any measure (AUM, volume traded, media chat).

History of VXX and VXZ since inception

- Less volatile VXZ
- More volatile VXX
Connecting the volatilities of the two products empirically

20-day regression coeff of daily returns: VXZ/VXX
Short the front-month ETN, long the back-month × 2 (since inception)

short 100% of VXX, long 200% of VXZ

Very profitable strategy until October 2010
The VIX futures curve is often far steeper in the near-term than in the far-term. As investors look to diversify their investment portfolios, money has poured into products that offer access to short-term VIX futures contracts, driving up contract prices significantly and further steepening the shorter end of the curve.

The UBS E-TRACS Daily Long-Short VIX ETN (UBS XVIX) offers sophisticated investors:

- The opportunity to capitalize on the steepness of the short end of the VIX futures curve
- A strategy that is historically uncorrelated to stock market returns
- Convenience of an exchange-traded security

The UBS E-TRACS Daily Long-Short VIX ETN is linked to the performance of the S&P 500 VIX Futures Term-Structure Index Excess Return (Ticker: SPVXTSER). The Underlying Index is a composite index that measures the return from taking a long 100% position in the S&P 500 VIX Mid-Term Futures™ Index Excess Return (the “Mid-Term Index”) with a short, or inverse, 50% position in the S&P 500 VIX Short-Term Futures™ Index Excess Return (the “Short-Term Index” together with the Mid-Term Index, the “Sub-Indices”), with daily rebalancing of the long and short positions.

The performance of the Underlying Index is reduced by the **Fee Amount of 0.85% per annum**, and increased by the Financing Payment.
VXX Options

- Options on VIX futures settle and are priced based on VIX futures (i.e. the “forward price” is the co-terminal VIX price). Black 76 works.

- What about options on VXX? **Contango plays a role** in the evolution of VXX prices, and thus the fair value of an option (regardless of volatility).

- Our point: The term-structure of VIX is volatile, so the forward pricing of VXX is not be that straight-forward.

- **Strategy A**: derive a better “physical measure” (or forecast) for VXX based on econometric analysis of VIX futures curves and roll formula.
  Trading strategy: buy cheap options and don’t delta hedge

- **Strategy B**: use the physical measure as prior and build a risk-neutral measure for trading VXX options relative to VIX options.
  Trading strategy: trade VXX options vs. VIX options (delta-neutral)

This paper discusses Strategy A

Joint work with Shishun Jiang, NYU
Build a good physical measure for VXX

- VXX is based on an index, which represents a theoretical roll between the two front contracts.

- Fit the term-structure of futures to a multivariate GARCH (1,1) model.

  Step 1: Perform PCA on the futures curve, parameterized by constant maturity

  Step 2: Derive significant Principal Components for VIX term-structure

  Step 3: Estimate the evolution of the Principal Component loadings as uncorrelated GARCH (1,1) processes

  This gives a tool for forecasting VXX.

For PCA: see Carol Alexander and Dmitris Korovilas (2002)
For GARCH on term structures: see Avellaneda and Zhou (1996)
Actual VXX is within the forecast prices (out of sample simulation)
Effect of the Downgrade of the US Treasury by S&P in August 2011

VXX simulations with 500 sample paths from 07–Jun–2011 to 17–Sep–2011

- VXX Market Price
- VXX Model Simulations
VXX Option Valuation

• **Market value:** this is the value of the VXX option from the market, on the pricing date.

• **Black Scholes with historical volatility:** based on an estimation window in the past, use the BS formula with historical volatility to determine a subjective price for VXX options on the pricing date.

• **New Model:** Use GARCH (1,1) estimation of the movements of the VIX futures curve to simulate scenarios of evolution of the VXX beyond the pricing date. Price options by averaging final payoff over GARCH statistics.

  Negative drift of VXX suggests opportunities in put options.
Price Date: June 7, 2010

Comparison of Market and Model VXX Put Prices

- Market Offer Price on 06/07/10
- Model Average Price
- Black-Scholes Model Price
- Model Price 16 Percentile
- Model Bid Price on 09/17/10
- Model Price 84 Percentile

Moneyness
Price Date: June 7, 2011

Comparison of Market and Model VXX Put Prices

- **Red Line**: Market Offer Price on 06/07/11
- **Blue Line**: Model Average Price
- **Green Line**: Black–Scholes Model Price
- **Dashed Blue Line**: Model Price 16 Percentile
- **Red Triangle**: Market Bid Price on 09/17/11
- **Dashed Blue Circles**: Model Price 84 Percentile

Moneyness

0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5

0 5 10 15 20 25 30 35
PNL analysis of Put-buying strategy (6/7/2010)
PNL analysis of Put-buying strategy 6/7/2011

Comparison of Market and Model VXX Put PnLs

- Market Realized PnL
- Model Average PnL
- Model PnL Standard Deviation
- Model PnL 16 Percentile
- Model PnL 16 Percentile Maximum = -0.49
- Model PnL 84 Percentile

Moneyness

99 Weeks, one Pricing Date per week.

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**Red = loss**

**White = profit**
Histogram of 99 trades
(Best trade selected on each date based on model)
X-Y Plot (signal strength/PNL)

Signal Strength = Expected PNL/Premium

PNL = Realized PNL / Option Premium
Histogram of best trades with signal threshold (model 16% quantile > 1.10; 49 trades)
Conclusion

• Commodity ETFs based on futures underperform the underlying commodity due to contango/backwardation

• Monetization of this effect outside the physical market can be done by pairing ETFs with different exposures along the futures curve

• VIX is a commodity and the VXX, VXZ ETNs are subject to contango effects.

• Opportunities in static portfolio building existed until 2010 all but disappeared, due to segmentation in the VIX curve

• Options on VXX and other ETNs still remain interesting, as they can be structured to monetize the dynamics of the slope of the futures curve.