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Recent Advances in Modeling Liquidity Risk and Applications to Central Clearing

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Outline

- Liquidity in risk management: listed markets, OTC Markets
- Liquidity-adjusted VaR for directional exposures
- BM&F Bovespa's *Close-out Risk Evaluation (CORE)*
- Modeling Liquidity Charges in OTC Markets & Liquidity Polls

Importance of Liquidity Modeling

- Need to go beyond VaR. In most cases, VaR is grossly insufficient due to prolonged, expensive liquidation.
- LTCM (1998): basis trades (long high spreads/short low spreads, DV01=0)
- Credit Crunch (2007, 2008): loss of liquidity for financing MBS, ABS and Credit Derivatives
- J.P. Morgan CIO (2012): basis trade, index CDS versus corporate CDS (2bb loss -> 8 bb loss)
- MF Global (2012): 2y term repos on Italian government bonds
- Grupo Interbolsa (2012) : Fabricato share repos, collapse of 2nd largest Colombian broker-dealer

Main Themes in Liquidity Modeling

<u>Liquidity = the ability to convert a given position into cash or risk-less securities</u>

- Size matters:

$$\Delta P = Const. \times |Q|^p$$

The larger the quantity, the worst is the outcome. p=1.5 is generally accepted, but there exists some controversy among academics, e.g. Gabaix p=0.5, Cont p=1

- Time-horizon matters: there is a maximum reasonable amount of inventory that can be liquidated in a given period of time (Almgren & Chriss, 2000).

  Break large orders into smaller pieces (TWAP, WVAP, PoV)

- Market structure matters: OTC derivatives markets have different price/liquidity discovery properties than listed markets.

Differences between OTC and Listed Markets

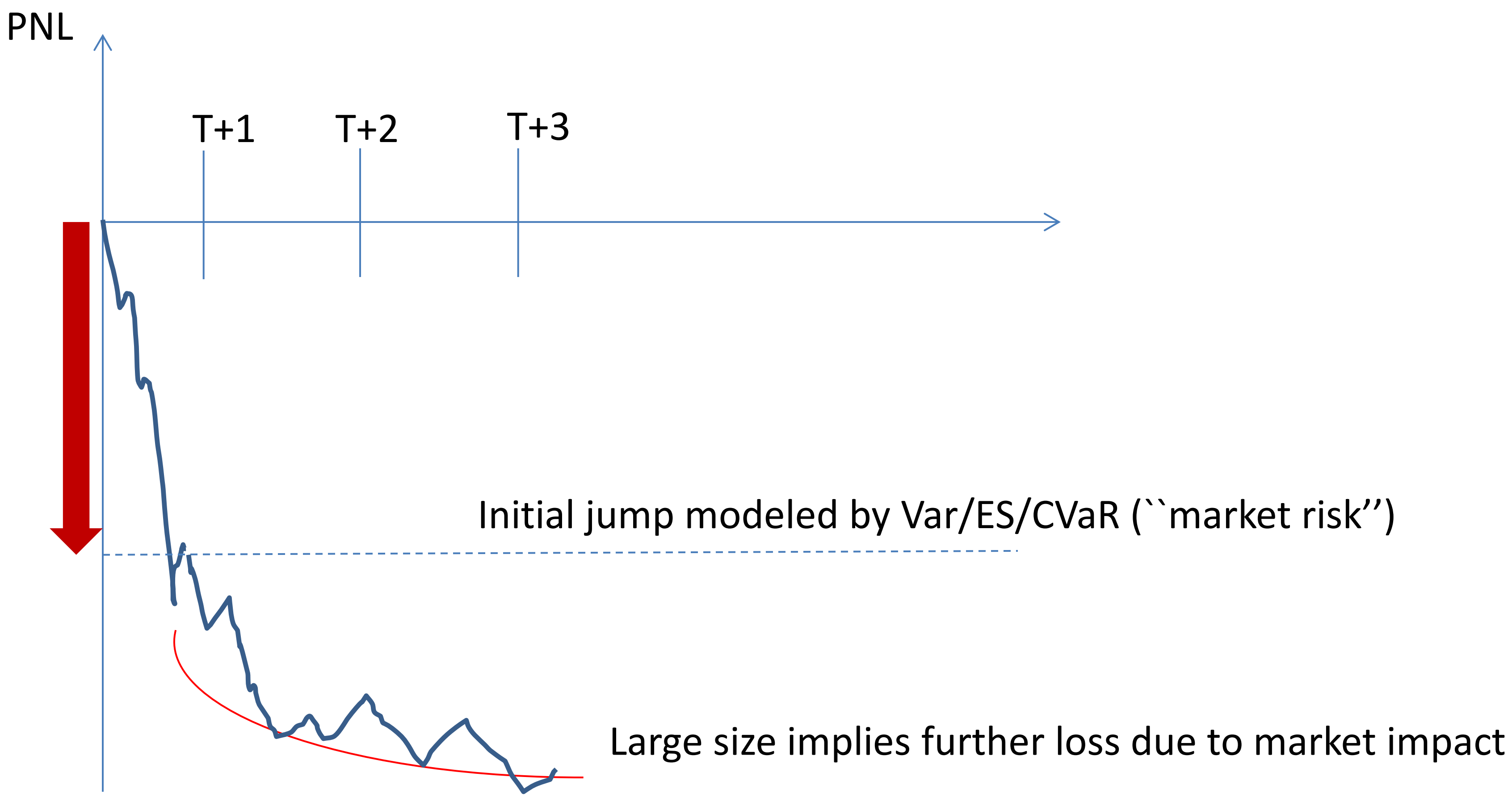
- In exchanges, liquidity models are based on daily trading volumes, open interest and bid-ask spreads.

- In over-the-counter (OTC) markets, trades and volumes are unknown and post-trade/market depth data are not easy to find (Avellaneda and Cont, 2010).

- Main challenge: to calculate suitable liquidity reserves or ``charges '' for portfolios.

- This problem is directly relevant to **central counterparty clearing** of derivatives and portfolio risk-requirements in the post-2008 era. In CCPs, liquidity must be modeled explicitly.

Liquidity-adjusted VaR: simple exercise or complex calculation?



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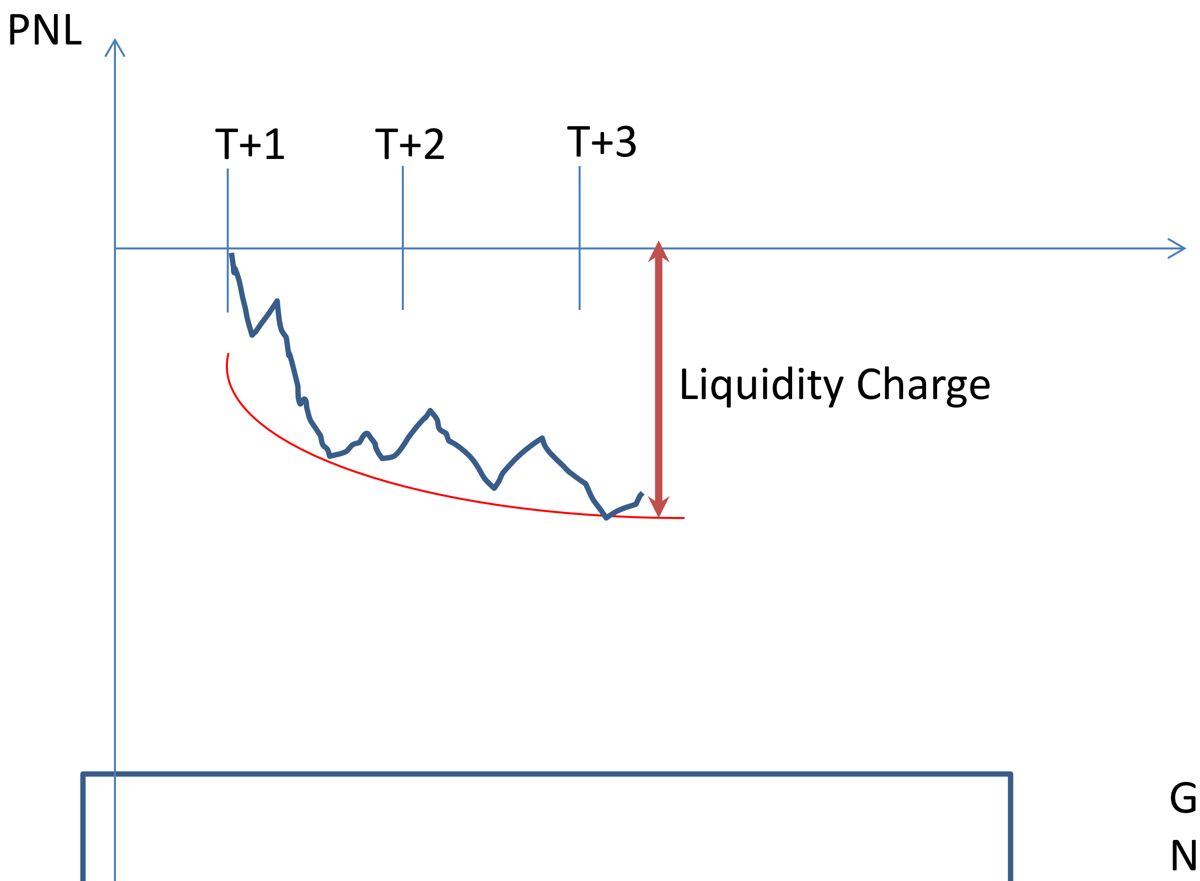
Heuristics: Liquidity Curves



$L(Q)$=a× $|Q|^{1.5}$

$$LVaR(Q \cdot X) = VaR(Q \cdot X) + a|Q|^{1.5}$$

Good for ``outright'' positions.
Not good for portfolios,
where we have offsets between
long and short positions in
instruments with common RFs

Simple liquidity model

Q = notional quantity
Q_0= ``typical'' trade size
σ = price volatility (%)
$N = Q/Q_0$ dimensionless size of the position
ξ_i = iid Student-t r.v. mean=0, variance=1, df >2

$$PNL(n) = \sum_{i=1}^{n} \sigma (Q - iQ_0) \xi_i$$

$$= \sigma Q \sqrt{N} \sum_{i=1}^{n} \left(1 - \frac{i}{N}\right) \frac{\xi_i}{\sqrt{N}}$$

$$= \sigma Q_0 \left(\frac{Q}{Q_0}\right)^{1.5} \sum_{i=1}^{n} \left(1 - \frac{i}{N}\right) \frac{\xi_i}{\sqrt{N}}$$

$$\approx \sigma Q_0 \left(\frac{Q}{Q_0}\right)^{1.5} \int_0^{\tau} (1 - t) dW_t \qquad N, n \gg 1, n/N \to \tau$$

Liquidity Add-on?

- Scaling gives a PNL for liquidation which is a standardized **stochastic process** (stochastic integral) multiplied by a nonlinear function of quantity.

- Any risk-measure (e.g. VaR, CVaR, STDEV) will give rise to a liquidation cost of the form

$$PNL = \kappa\, \sigma\, Q_0 \left(\frac{Q}{Q_0}\right)^{1.5} = \kappa\, \sigma \frac{Q^{1.5}}{{Q_0}^{0.5}}$$

- This is the asymptotics for large Q. Therefore, the LC should be something like this

$$LC(Q) = \kappa\, \sigma\, Q_0 max\left[\left(\frac{Q}{Q_0}\right)^{1.5}, \frac{Q}{Q_0}\right] \qquad \kappa\sigma \cong \frac{1}{2}\text{(bid-ask spread)}$$

Example: Eurodollar Futures

- CME ED Futures: Median Volume ~ 100 K contracts
- Tick size (and bid/ask spread)= 0.25 bps for front month, 0.5 bps others.
- Contract sensitivity = USD 25 per LIBOR basis point move
- Reasonable volume = 10% daily volume = 10,000 contracts
- Typical liquid (large) trade = 10,000$\times$ $25 = 250{,}000$ USD of DV01

Deriving Liquidity Curves

- Liquidity Charge for normal trades= ½ bid-ask spread

Front month = 1/8 basis point
Out months = ¼ basis point

$$LC(X) = \frac{1}{2} s \times \left(\frac{X}{X_0}\right)^{1.5}$$

$$LC_f(X) = (0.125)\left(\frac{X}{0.25}\right)^{1.5} = X^{1.5}$$

$$LC_b(X) = (0.25)\left(\frac{X}{0.25}\right)^{1.5} = 2\,X^{1.5}$$

	DV01			
spread	1 MM	5 MM	10 MM	25 MM
0.25	1	11	32	125
0.5	2	22	63	250
1	4	45	126	500

Liquidity charges in basis points of notional
For different sizes measured in MM Dv01

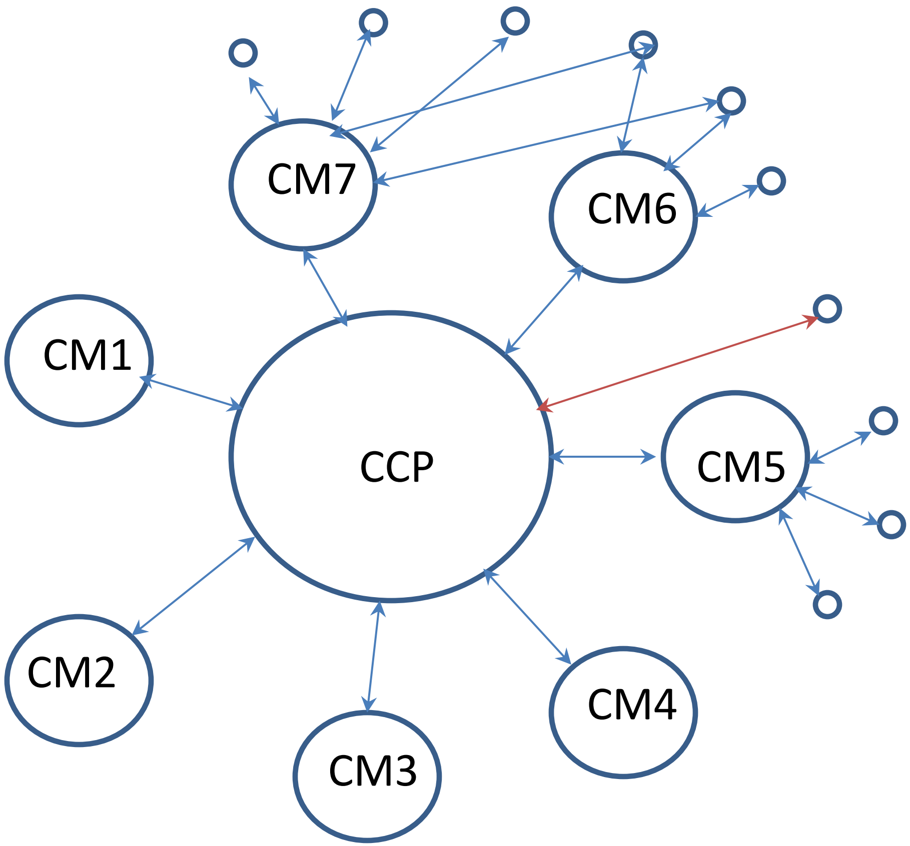
Market and Liquidity Risk for Portfolios

Central clearing & risk-management

Large circles:
Clearing members
(large banks, BDs)

Small circles:
non-clearing market participants (buy side)

Arrows represent credit exposure



Major Clearinghouses Today

Inter-bank payments: ACH

Securities: DTCC, FICC, LCH.Clearnet, Eurex Clearing

Derivatives: CME Group, LCH.Clearnet, Intercontinental Exchange (ICE), ICE Clear Europe, Eurex Clearing, Hong-Kong Exchanges and Clearing

Equity Options: The Options Clearing Corporation

In Brazil, BM&F Bovespa manages 4 CCPs for different asset classes (like CME Group, which clears commodities, financials, and some OTC)

Tools for Risk Management of CCPs

- Initial Margin (Market Risk, **Liquidity Risk**)
- Fund for mutualization of losses (``Guarantee Fund''), covering shortfall beyond the IM
- ``Loss tranches'' with CCP's own capital (Skin in the game)

BIS, Principles for Financial Markets Infrastructures, BIS CPSS-IOSCO Consultative Report, April 2012

ESMA, Final Report, Technical Standards on OTC Derivatives, CCPs and Trade Repositories (``EMIR''), September 2012

Some CCP Risk-management issues which combine market and liquidity risk

- How can the CCP remain well-capitalized during the liquidation of a defaulted participant?
- Create synergies by using a common system to clear different products with the same risk factors (e.g. listed and OTC derivatives, collateral-in-margin)?
- How to handle a portfolio of securities sensitive to the same risk-factors but having different liquidity ?
- Treat portfolios which have daily settlement (futures) as well as OTC securities (forwards) which do not have daily settlement?

Close Out Risk Evaluation (CORE)

Close-Out Risk Evaluation (CORE) proposed by BM&F Bovespa

- Find a suitable liquidation strategy for each participant's portfolio
- Compute potential uncollateralized losses associated with liquidation of the portfolio under stress scenarios
- Margin requirement is based on potential losses over the liquidation period

BM&FBOVESPA's Post-Trade Infrastructure: Integration Opportunities and Challenges, September 2010, www.bmf.com.br/bmfbovespa/pages/boletim2/informes/2010/marco/WhitePaper.pdf

Example 1: Liquid vs. illiquid stock

Petrobras average trading volumes
PETR4: 24 million shares/day (assume max liq. =3MM)
PETR3: 8 million shares/day (" =1MM)

<u>Portfolio: Long 40 MM PETR4, short 30MM PETR3</u>

Day	1 5 30
PETR4 (sell)	3 3 3 3 3 1 1 1 1 1 1 1 1 ----------------------- 1 1 1 1 1 1
PETR3 (buy)	1 1 1 1 1 1 1 1 1 1 1 1 1 ----------------------- 1 1 1 1 1 1

Mkt. exposure: 10 MM PETR4 for 5 days
PETR 4 ``hedges'' PETR 3 MTM P/L during liquidation

Liquidating independently of common risk factors (naïve liquidation)

Day	1 5 14 30
PETR4 (sell)	3 3 3 3 3 3 3 3 3 3 3 3 3 1 0 0----------------------- 0 0 0
PETR3 (buy)	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1----------------------- 1 1 1

Mkt. exposure: 26 MM PETR4 for first 13 days, 16 MM PETR3 for last 15 days
Much more market risk than previous example.

Naïve liquidation always costs more. Some optimization can be done.

Example 2: portfolio of futures and OTC forwards with bond collateral

Portfolio:

Long 10,000,000 BRL in USD futures (liquid)

Short 10,000,000 BRL in forwards (illiquid, with auction in 10 days)

Long 20,000,000 in Brazilian T-bills (LTN) (liquid, but not cash)

Naïve Strategy #1:

- Close futures position, sell T-bills, wait 10 days with the forward position

Naïve Strategy #2:

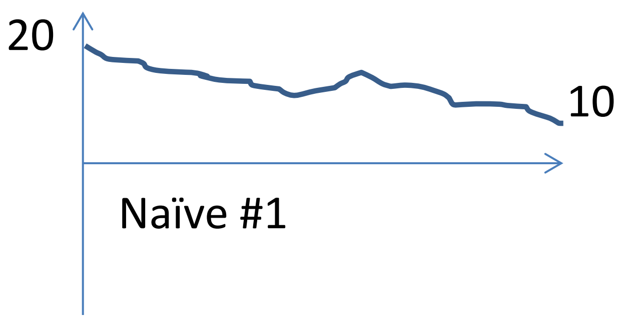
- Do not close futures position, wait 10 days with forward position, then sell T-bills

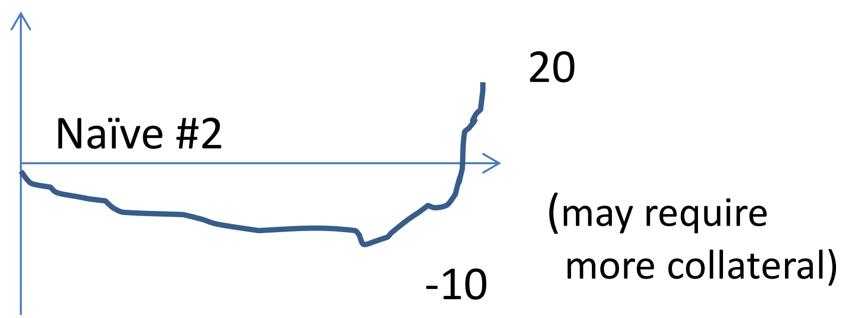
Correct strategy takes into account daily settlement of futures

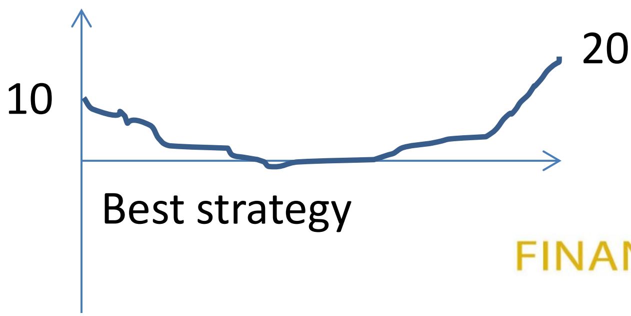
<u>Best strategy:</u>

- Do not close futures position, wait 10 days with forward position,

  Sell a fraction of T-Bills to cover variation margin in Futures for 10 days.
  Close the remaining portfolio in 10 days







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Modeling portfolios with liquidity constraints

- In a world with infinite liquidity, a portfolio is represented as a list of instruments and quantities

DOL Fut 01/2013	VALE5	GUAR3	BOVA11	IBOV Fut 04/2013
2,000	-45,000	53,000	-20,000	3,000

- In a world with limited liquidity, we should include the maximum amounts that can be traded in a given period (day) without `moving the market'*

DOL Fut 01/2013	VALE5	GUAR3**	BOVA11	IBOV Fut 04/2013
2,000	-45,000	53,000	-20,000	3,000
25,000	1,000,000	1,000	150,000	10,000

* Proxied here at 10 % Avg. Traded Volume

** Guararapes Confecc. SA

Portfolio Description

MTM_1(t,R)	MTM_2(t,R)	MTM_3(t,R)	MTM_4(t,R)	MTM_5(t,R)
Q_1	Q_2	Q_3	Q_4	Q_5
l_1	l_2	l_3	l_4	l_5

- R represents the state of the market or path of states of the market (risk-factor changes)

$$\boldsymbol{R} = (R_0, R_1, R_2, \dots R_t, R_{t+1}, \dots)$$

- Example: if we are dealing with options, then $R_t = \begin{pmatrix} S_t \\ \sigma_t \\ r_t \\ d_t \end{pmatrix}$

S_t	Und. Price
σ_t	Volatility
r_t	Interest rate
d_t	Dividend yield

The Risk-factors

- Q_i,l_i represent quantities and daily liquidity limits for each instrument

Liquidation of a Portfolio: `Close-out strategy'

- On date t=0, you decide that a portfolio should be liquidated starting on t=1.

- Determine a strategy in which a certain fraction, q_{it}, of the of the position in instrument *i* will be liquidated at date *t*. ($q_{it}, i = 1, \ldots, N, t = 1, \ldots, T_{max}$)

$$\begin{cases} 0 \leq q_{it} \leq \frac{l_i}{Q_i} \equiv k_i \qquad \forall i \; \forall t \\ \sum_{t=1}^{T_{max}} q_{it} = 1 \end{cases}$$

$$n_t = \sum_{s=t+1}^{T_{max}} q_s$$

The remaining balance (%) at time t

- A close-out strategy is a matrix that tells us how to proceed for liquidating the various instruments in the portfolio as time passes.

Defining the objective function: Profit and loss of a close-out strategy for a portfolio

$$\psi_i(t, R_t) \stackrel{\text{def}}{=} Q_i[MTM_i(t, R_t) - MTM_i(0, R_0)]$$

P/L, full valuation

- Realized P/L at date t, after trading

$$L_r(t, q, R_t) = \sum_{i=1}^{N} q_{it}\psi_i(t, R_t)$$

- Unrealized (a.k.a. MTM) P/L at date t, after trading

$$L_{nr}(t, q, R_t) = \sum_{i=1}^{N} n_{it}\psi_i(t, R_t)$$

Accumulated P/L

- Accumulated profit/Loss for close out strategy at date t

$$L(t, q, R\ ) = \sum_{s=1}^{t} L_r(s, q, R_s) + L_{nr}(t, q, R_t)$$

$$= \sum_{s=1}^{t} \sum_{i=1}^{N} q_{is} \psi_i(s, R_s\ ) + \sum_{i=1}^{N} n_{it} \psi_i(t, R_t\ )$$

cash unrealized gain/loss

CORE objective function

- Define scenarios for the risk-factors, $R = \left(R_1, R_2, \ldots, R_{T_{max}}\right)$
- These scenarios are paths. Let $\boldsymbol{R}$ denote the set of all scenarios considered

$$U(q) = \min_{R \in \boldsymbol{R}} \min_{1 \le t \le T_{max}} L(t, q, R)$$

$$= \min_{R \in \boldsymbol{R}} \min_{1 \le t \le T_{max}} \left(\sum_{s=1}^{t} \sum_{i=1}^{N} q_{is} \psi_i(s, R_s) + \sum_{i=1}^{N} n_{it} \psi_i(t, R_t) \right)$$

The Optimization Problem

Maximize $U(q) \quad q = (q_{it}) \in R^{N \times T_{max}}$

Subject to:
$$\begin{cases} 0 \le q_{it} \le \frac{l_i}{Q_i} \equiv k_i & \forall i \ \forall t \\ \sum_{t=1}^{T_{max}} q_{it} = 1 & \forall i \end{cases}$$

- U(q) is a sum of minima of linear functions of q => it is concave
- The set of constraints is convex (it is a convex polyhedral region)

A solution exists and should be unique under reasonable conditions!

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Solution Via Linear Programming for U_2

Maximize: U

Variables: $\{U, \lambda_t, \mu_t, q_{it}; 1 \le t \le T_{max}, 1 \le i \le N\}$

Subject to constraints:

$$U \le \lambda_t + \mu_t \qquad \forall t$$

$$\lambda_t \le \sum_{i=1}^{N} q_{it}\psi_i(t, R_t) \qquad \forall t\ \forall R \in \boldsymbol{R}$$

$$\mu_t \le \sum_{i=1}^{N} (q_{it} + n_{it})\psi_i(t, R_t) \qquad \forall t\ \forall R \in \boldsymbol{R}$$

$$0 \le q_{it} \le \frac{l_i}{Q_i} \equiv k_i \qquad \forall i\ \forall t$$

$$\sum_{t=1}^{T_{max}} q_{it} = 1 \qquad \forall i$$

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BMF-CORE Liquidity Adjusted Risk Margin

$$M = \max_q \min_{R\epsilon\boldsymbol{R}} \left(\min_{1 \le t \le T_{max}} L(t, q, R) \right)$$

$$= \min_{R\epsilon\boldsymbol{R}} \left(\min_{1 \le t \le T_{max}} L(t, q^*, R) \right)$$

q^* = optimal close-out strategy

- **Alternative versions**, which can be used with Monte Carlo models for RFs,

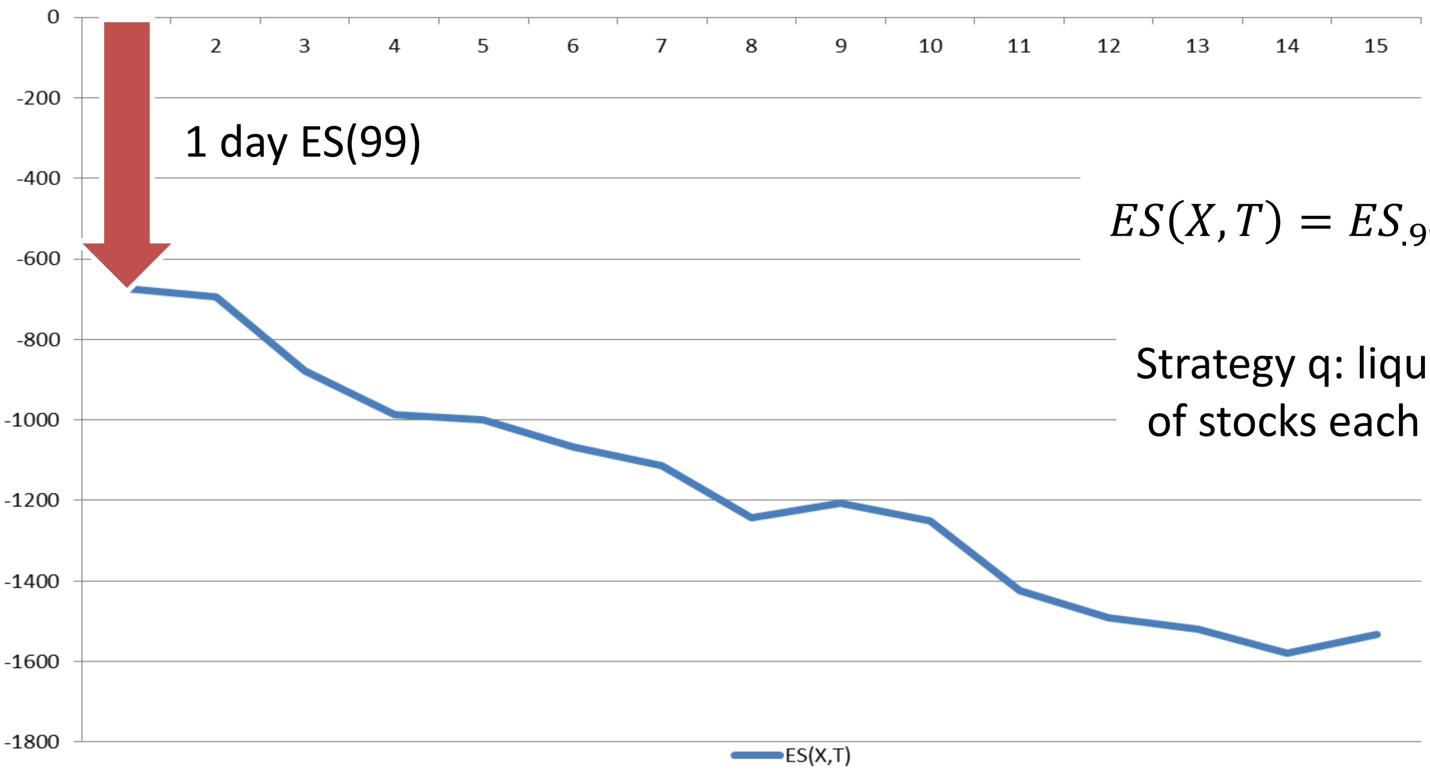
$$VaR_\alpha \left(\min_{1 \le t \le T_{max}} L(t, q^*, R) \right) \qquad \alpha = .99, or\ .995$$

$$ES_\alpha \left(\min_{1 \le t \le T_{max}} L(t, q^*, R) \right) \qquad \alpha = .99 \ldots$$

Example: Liquidation of a portfolio of stocks using the CORE risk-measure and Historical Monte Carlo

- 10 stocks, 100 shares per stock

SPY	GDX	UVXY	VTI	VWO	SIL	TLT	IWM	AGG	VOO



$$ES(X,T) = ES_{.99}\left(\min_{t \leq T} L(t,q,R)\right)$$

Strategy q: liquidate equal amounts of stocks each day (not optimal)

Portfolio Liquidity Modeling in OTC Markets

Liquidity in OTC Risk-Management

- In OTC risk-management, the portfolio is liquidated in an auction (there is not exchange).
- Participants periodically inform the CCP on liquidity and market depth, so IM requirements can take into account liquidation costs.
- Difficulties may arise since the ex-ante liquidation costs used for risk management are provided by agents which will bid on the defaulted portfolio at auction (ex-post).
- The time dimension of liquidation should be ``made equivalent'' to a wider B/O spread in a 1-day auction.

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Polling

- Polls are conducted asking CCP participants by how much would their bid or ask price change as a function of trade size

- Liquidity polls typically involve

  -- directional positions

  -- market-neutral portfolios

- Liquidity charges for market-neutral portfolios are typically lower than for outright positions because they have less risk exposure

- To some extent, the poll incorporates the time-dimension of the close-out process

Example: Swaps

Tenor (yrs)	Swap positions (in MM DV01) 2	5	10	30
Port 1	1			
Port 2		1		
Port 3			1	
Port 4				1
Port 5	1	-1		
Port 6	1		-1	
Port 7	1			-1
Port 8		1	-1	
Port 9			1	-1
Port 10	1	-2	1	
Port 11		1	-2	1
Port 12	1	-2		1
Port 13	1		-2	1

- We consider 4 standard swap tenors. A typical poll will consider several portfolios: outright swap positions, curve positions (or time spreads) and butterfly spreads. <u>Portfolios 5 to 13 are market-neutral.</u>

Liquidity Charge Curves obtained by polling 10 dealers and taking median values

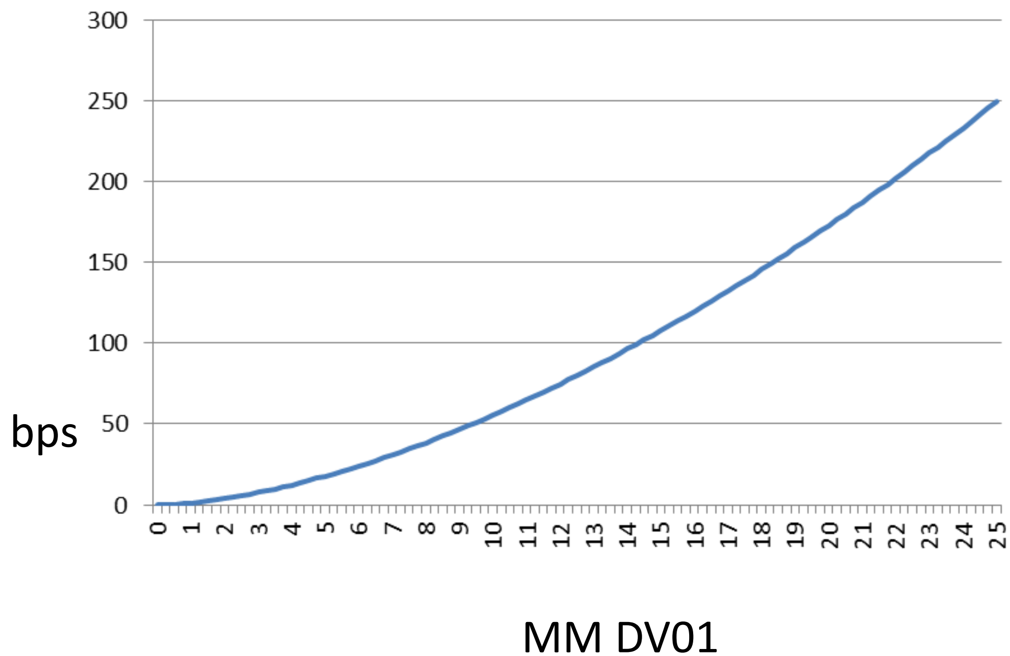
Portfolio	multiplier (1X, 5X,10X,25X)			
	1	5	10	25
1	1.25	17.50	60.00	237.50
2	1.93	20.00	70.00	250.00
3	2.25	25.00	80.00	337.50
4	3.00	30.00	100.00	450.00
5	1.00	12.50	45.00	168.75
6	2.00	18.18	60.00	212.50
7	2.00	20.61	80.00	275.00
8	2.00	15.00	47.50	187.50
9	2.00	20.00	60.00	253.13
10	2.00	20.00	80.00	-
11	2.00	30.00	80.00	-
12	4.00	40.00	150.00	-
13	4.00	40.00	135.00	-

- We use the median value as an indicator of the function F(N) for each spread
- Rows 1 and 2 are similar to what we obtained earlier for ED futures based on 1.5 model

Smoothing the Poll Data

- Take the discrete poll and fit the data to power-laws using log-log regression

Outright 2Y swap



$$F(N) = (1.26827) * N^{1.6406}$$

Bps for 1MM DV01

Exponent greater Than 1

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Empirical results for LCCs

Liquidity Charge Curves (in bps per USD 1 million DV01

Spread	ln a	a	b	R-squared
1	0.23765327	1.268269369	1.640640912	0.990944737
2	0.638018832	1.892727352	1.524495236	0.979830862
3	0.7800459	2.181572398	1.558456377	0.993419153
4	1.030187386	2.801590763	1.555690894	0.972527236
5	7.46475E-05	1.00007465	1.607219352	0.983037657
6	0.65997447	1.93474294	1.45941347	0.981681918
7	0.667487666	1.949333786	1.549304728	0.958037632
8	0.608367126	1.837428659	1.411071285	0.956494481
9	0.652015481	1.919405458	1.501764209	0.990402548
10	0.640578078	1.897577509	1.571858782	0.951821306
11	0.717869273	2.050060434	1.616243426	0.989358386
12	1.342315061	3.827895065	1.548779256	0.966291357
13	1.356352854	3.882009196	1.511079491	0.984376

Table 1: Results of log-log regression for the 13 spreads in the swaps example, using median poll values. The fit is to the curve $F = aN^b$ which is done by estimating the linear model $lnF = lna + b\,(lnN) + \epsilon$. The R^2 coefficient is given in the right-most column. Notice that the empirically estimated exponents b are reasonably close to the b=1.5 heuristic value.

Liquidity Add on
Charge Calculation for IR Swap Portfolios

- Represent swap portfolios as loadings on the standard tenor swaps (2y,5y,10y,30y)

- Minimize:

$$\sum_{i=1}^{N} F_i(Q_i)$$

subject to the linear constraints

$$\sum_{i=1}^{N} \mu_{im} Q_i = \Delta_m \qquad m = 1, \dots, M$$

and bound constraints

$$|Q_i| \leq Q_{max,i} \qquad i = 1, \dots, N$$

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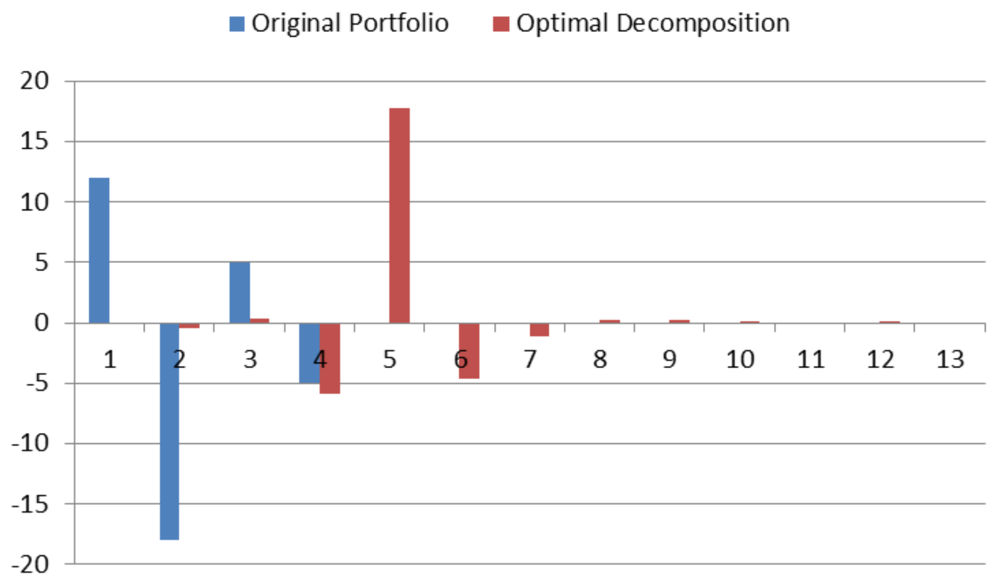
Example

TARGET PORTFOLIO		(in MM DV01)		
TENOR	2	5	10	30
DV01 MM	12	-18	5	-5
CHARGE (bp	74.78	155.15	22.01	34.26
NAÏVE CHG	**286.20**			

	CALCULATED	CALCULATED
SPREAD	HEDGE POSITION	LIQUIDITY CHARGE
1	0.00	0.00
2	-0.46	0.59
3	0.33	0.38
4	-5.86	43.86
5	17.72	101.57
6	-4.64	18.14
7	-1.09	2.22
8	0.20	0.19
9	0.22	0.20
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
	Sum of charnges	167.15
	NAÏVE CHARGE	286.20
	SMART CHARGE	167.15



Exp Shortfall 99% 234,136,074 (Margin)

Liquidity Charge 167,145,075

Conclusions

- Liquidity Modeling is an integral part of risk management.
- In CCPs, liquidity is essential for constructing a sound margin system for portfolios.
- Models should include portfolio size and time-horizon (model the close-out!)
- *CORE* (BMF&F Bovespa) suggests constructing *liquidity thresholds* for each security and finding a liquidation strategy so that the worst value of the portfolio along the liquidation period under stress scenarios is optimized
- In OTC markets, where prices are contributed by participants, liquidity polls for reference portfolios at different trade sizes can be used to build liquidity curves.
- There is some indication that poll-based estimates are consistent with the CORE approach. Empirically we found that LCCs from polls are convex functions of trade size and appear to follow the *``1.5 model''.*