

Derivative Securities, Fall 2011

Assignment 2, Due October 19th 2011, 7PM

1. Give concrete examples of exchange-traded options markets where derivatives are (i) American-style and (ii) European-style.

2. US stocks usually pay dividends quarterly. There are several dates associated with each dividend distribution. (1) The dividend announcement date, (2) the *ex-dividend date* (3) the dividend payment date. The *ex-dividend date* is the most important: it is the last date for which stock-holders are entitled to collect the dividend. Usually, the stock price drops by an amount equal to the dividend after the close of the *ex-dividend date*. Why? Show that, from a logical point of view (and if the stock is not hard-to-borrow), it makes sense to exercise an American call *only* on the *ex-dividend date*.

3. Using the following tables, calculate the implied dividend yield for the Jan 2012 and April 2012 ATT options.

T= 28.55	Expiration Date:		20-Jan-12			
Calls				Puts		
Symbol	Bid	Ask	Strike	Symbol	Bid	Ask
T120121C00015000	13.45	14.25	15	T120121P00015000	0.06	0.08
T120121C00017500	11	11.75	17.5	T120121P00017500	0.13	0.15
T120121C00020000	8.55	8.6	20	T120121P00020000	0.25	0.26
T120121C00022500	6.05	6.1	22.5	T120121P00022500	0.44	0.46
T120121C00024000	4.65	4.7	24	T120121P00024000	0.63	0.66
T120121C00025000	3.75	3.85	25	T120121P00025000	0.81	0.84
T120121C00026000	2.97	3.05	26	T120121P00026000	1.05	1.08
T120121C00027500	1.92	1.95	27.5	T120121P00027500	1.56	1.58
T120121C00029000	1.09	1.12	29	T120121P00029000	2.29	2.32
T120121C00030000	0.7	0.71	30	T120121P00030000	2.91	2.95
T120121C00031000	0.4	0.42	31	T120121P00031000	3.6	3.7
T120121C00032500	0.15	0.17	32.5	T120121P00032500	4.9	5
T120121C00034000	0.04	0.06	34	T120121P00034000	5.8	6.45
T120121C00035000	0.02	0.03	35	T120121P00035000	7.25	7.35
T120121C00036000	N/A	0.05	36	T120121P00036000	7.75	8.4
T120121C00037500	N/A	0.01	37.5	T120121P00037500	9.25	9.9
T120121C00040000	N/A	0.03	40	T120121P00040000	11.65	12.45
T120121C00042500	N/A	0.03	42.5	T120121P00042500	14.1	14.9

Expiration: April 20,2012,
T=28.55

Calls				Puts		
Symbol	Bid	Ask	Strike	Symbol	Bid	Ask
T120421C00020000	8.55	8.6	20	T120421P00020000	0.48	0.5
T120421C00023000	5.65	5.7	23	T120421P00023000	0.91	0.94
T120421C00024000	4.75	4.85	24	T120421P00024000	1.13	1.15
T120421C00025000	3.95	4.05	25	T120421P00025000	1.39	1.41
T120421C00026000	3.25	3.35	26	T120421P00026000	1.7	1.73
T120421C00027000	2.59	2.63	27	T120421P00027000	2.08	2.12
T120421C00028000	2	2.05	28	T120421P00028000	2.53	2.57
T120421C00029000	1.5	1.54	29	T120421P00029000	3.05	3.1
T120421C00030000	1.08	1.12	30	T120421P00030000	3.65	3.75
T120421C00031000	0.75	0.79	31	T120421P00031000	4.35	4.45
T120421C00032000	0.5	0.54	32	T120421P00032000	5.1	5.2
T120421C00033000	0.32	0.36	33	T120421P00033000	5.95	6.05
T120421C00034000	0.2	0.23	34	T120421P00034000	6.35	7.05
T120421C00035000	0.12	0.14	35	T120421P00035000	6.95	8.05

Discuss how these implied dividends are compatible with the following distribution table for ATT;

Div	Ex-Date
0.43	Jul 6 2011
0.43	Apr 6 2011
0.43	Jan 6 2011
0.42	Oct 6 2010
0.42	Jul 7 2010
0.42	Apr 7 2010
0.42	Jan 6 2010

(The data is also online in my webpage in Excel format, for your convenience.)

- Derive the Black-Scholes formula for a European-style put from the formula for a call using put-call parity.
- Put-call symmetry.** Assume that an asset has dividend yield=0 and that interest rate is also zero. Assuming a standard Black-Scholes model for the pricing measure, show that if $F / K^* = K / F$ where F is the forward price, we have

$$Put(S, K^*, T) = \left(\frac{F}{K} \right) \cdot Call(S, K, T).$$

Try to see how well this applies to the Jan 2012 ATT's near the money.

- Mid-point rule.** Argue that if $K = F_{0,T}$, (the forward price) then, we have the approximate relation

$$Call(S, K + \Delta K, T) \cong Call(S, K - \Delta K, T) + \Delta K.$$

(Hint: use Taylor expansion about K, and estimate the value of Delta, the derivative of the call at F=K). State clearly the assumptions under which this should hold and the approximation error. Derive a more accurate relation between the values of calls with consecutive strikes at a fixed maturity using Taylor expansion in ΔK , and $F / K - 1$.

Test the above approximation in practice for SPY Dec 2011 calls (Yahoo! Finance).

- Show that if $K = F_T$ (the forward price of the asset) then

$$Call(F_T, T) \cong e^{-rT} \frac{F_T}{\sqrt{2\pi}} \sigma \sqrt{T} = e^{-rT} 0.4 \cdot F_T \cdot \sigma \sqrt{T}, \text{ valid for } \sigma \sqrt{T} \ll 1.$$

Test this approximation in practice for SPY options with April 2012 and June 2012 maturities from Yahoo! Finance.

For convenience of grading, please submit your assignment as a hard copy. Also, it's nicer for us if the assignments are typed. Thanks!