**Homework 4, Derivative Securities, Due December 7th**

1. Consider a 6-months up-and-out call option on an index with spot price S=$110, strike price=$100 and knock-out barrier H=$120. Assume that r=0.1%, a dividend yield of 4% and a volatility of 20%. Price this option using the analytic solution in Hull's book. Re-value using $\sigma = 10\%, 15\%, 25\%, 30\%$. Explain the results.

2. Build a finite difference scheme (trinomial tree or lattice), consistent with the above parameters. Re-price the above option with the same volatility values and thus verify that your scheme is correct.

3. (Term structure). We will now price the option assuming that volatility is not constant, but rather, that there is a forward volatility function $\sigma_t$. This function is assumed to satisfy the equation

$$\sigma_t{}^2 = 0.02 + t.0.08, \qquad 0 < t < 0.5 \qquad (1)$$

   Recalling that the relation between term and forward volatilities is given by

$$\bar{\sigma}_T^2 = \frac{1}{T}\int_0^T \sigma_s^2\, ds$$

   Verify that the term volatility in (1) is 20%. Then, use the trinomial scheme to compute the value of the barrier option assuming (1). What is you conclusion? Is the fair value computed in this way bigger or smaller than the one found in 1.? Why?

4. Same problem with a downward-sloping forward volatility curve

$$\sigma_t{}^2 = 0.06 - t.0.08, \qquad 0 < t < 0.5$$

   [Hint: Draw a lot of pictures!]

5. ( Monte-Carlo) Simulate lognormal paths of the index by setting

$$\ln(S_{n\Delta t+\Delta t}) = \ln(S_{n\Delta t}) + \left(r - q - \frac{\sigma^2}{2}\right)\Delta t + \sigma \nu_n \sqrt{\Delta t}$$

   where $\Delta t = \frac{1}{360}$, $r = 0.001$, $q = 0.04$, $\sigma = 0.2$ and $\nu_n$ are i.i.d. N(0,1) random deviates. Using 10,000 simulated paths, price the barrier option in 1 by calculating the payoff along each path, discounting, and averaging. Compare the answer with 1. We expect that the Monte Carlo method gives close answer but is biased slightly higher. Can you guess why this would be the case? [Hint: discretization.]