1. Show that $\zeta(0) = -1/2$ and $\zeta'(0) = -\frac{\ln(2\pi)}{2}$.

2. Show that for $s \in \mathbb{R}$, $\zeta(s) = 0$ if and only if $s = -2k$, with $k \in \mathbb{N}$, $k \geq 1$.

3. Let $f(x_1, \ldots, x_n) = \sum_{i=1}^{n} a_i x_i^m$, with $a_i \in \mathbb{Z}_p, a_i \neq 0$. Put

$$r := \nu_p(m), \ s := \max_i (\nu_p(a_i)) \quad \text{and} \quad N := 2(r + s) + 1.$$ 

Show that $f(x_1, \ldots, x_n)$ has a nontrivial zero in $\mathbb{Q}_p$ if and only if the congruence $f(x_1, \ldots, x_n) \equiv 0 \mod p^N$ has a solution $(x_0^1, \ldots, x_0^n)$ such that $x_0^j \not\equiv 0 \mod p$ for at least one $j$.

4. Show that $2B_{2k} \equiv 1 \mod 4$, for $k > 1$.

5. Check that $B_{2k} \not\equiv 0 \mod 17$ for $k = 1, \ldots, 7$, and that there exists a $k \in \{1, \ldots, 17\}$ such that $B_{2k} = 0 \mod 37$. 