Capital Markets and Portfolio Theory, Fall 2001

Solution Set 4

1. You need to integrate the differential equation \(-u''(w)/u'(w) = c\), which you can rewrite as \(u''(w) + cu'(w) = 0\), to obtain the general solution \(u(w) = ke^{-aw}\). Here \(k\) and \(c\) are arbitrary constants which parameterize the utility functions with the given property.

2. In each case you need to calculate the first and second derivatives and then use the definition:

   (a) \(R_{1,a} = \frac{b}{1-bw}\), \(R_{1,r} = \frac{bw}{1-bw}\). Obviously \(R_{1,a}\) is increasing, hence we have increasing risk aversion.

   (b) \(R_{2,a} = \frac{2}{w}\), \(R_{2,r} = -\gamma\). Decreasing risk aversion.

   (c) \(R_{3,a} = a\), \(R_{3,r} = aw\). Constant risk aversion.

3. It is easy to calculate that the function \(u(w) = 1, 255w - 0.5w^2\) achieves its maximum at \$1, 255; also \(u(100) = 755, 000 > u(2000) = 510, 000\). Then for the risk aversion we have \(R_a = \frac{1}{2250 - w}\), which obviously increases with \(w\). The investor is increasingly risk averse and will take less risk.

4. We have the following:

   (a) We do not know anything about the investors’ overall utility function, or about their wealth, so we can not say anything about how much of any asset each one is holding.

   (b) Although the utility functions are the same they still may have different wealth, so we do not know how much of risky asset they have.

   (c) Two utility functions with the same risk aversion differ by an affine linear transformation, so the choice does not affect the investment decisions.
5. We have the following calculations:

(a) First we have

\[
R_a = \begin{cases} \frac{1}{w}, & \text{if } w \leq c \\ 0 & \text{if } w > c \end{cases} \quad R_r = \begin{cases} 1, & \text{if } w \leq c \\ 0 & \text{if } w > c \end{cases}
\]

(b) The four possible outcomes are:
1. House burns & appreciates 40% in value; \( p_1 = 0.001 \cdot 0.5; \$50K \)
2. House burns & current value; \( p_2 = 0.001 \cdot 0.5; \$50K \)
3. House does not burn & appreciates; \( p_3 = 0.999 \cdot 0.5; \$400K \)
4. House does not burn & current value; \( p_4 = 0.999 \cdot 0.5; \$300K \)

6. Let \( w_i, i = 1, \ldots, 4 \) be the wealth of Mr. Pink in each possible outcome. The expected utility with no policy is \( E(u(w)) = \sum_{i=1}^{4} p_i u(w_i) \).

Let \( w_i^A, i = 1, \ldots, 4 \) be the wealth of Mr. Pink in each possible outcome when he has policy A, which costs \( p_A \). Its expected utility is

\[ E_A(u(w)) = \sum_{i=1}^{4} p_i u(w_i^A - p_A) \]

Obviously Mr. Pink is willing to pay up to \( p_A \), so that

\[ \sum_{i=1}^{4} p_i u(w_i) = \sum_{i=1}^{4} p_i u(w_i^A - p_A) \]

We have \( c = 1,000,000 \) and then obtain \( p_A = 613 \). In the same way we obtain \( p_B = 662 \).

7. We have \( c = 400,000 \) and then obtain \( p_A = 613 \), and \( p_B = 662 \).

8. In this case \( c = 500,000 \) and Mr Pink inherits \$1,000,000. We have \( p_A = 250 \) and \( p_B = 300 \).