Capital Markets and Portfolio Theory, Fall 2001

Solution Set 3

1. We use the results from the previous homework.

   (a) The first portfolio consists of a 10mm 7 year bond with \( c = 5\% \) \( y = 4.75\% \) and a 7.13mm 10 year bond with \( c = 6.25\% \) and \( y = 4.9\% \). The current value is \( P(0) \approx $2,254,340 \). For a parallel shift of 100bp up we have \( P = 10mm(0.95727) - 7.13mm(1.02616) \approx $2,247,689 \), so \( P(+100bp) - P(0) = 6,650.68 \). For a parallel shift of 100bp down we have \( P = 10mm(1.0763) - 7.13mm(1.1931) \approx $2,246,847 \) so we have \( P(-100bp) - P(0) = -$7,492.27 \).

   (b) The second portfolio consists of a 10mm 7 year bond with \( c = 5\% \) \( y = 4.75\% \), a 2.926mm 10 year bond with \( c = 6.25\% \) and \( y = 4.9\% \) and a 7.84mm 5 year bond with \( c = 4.4\% \) and \( y = 4\% \). We have \( P(0) \approx -$1,070,124 \). Then \( P(+100bp) - P(0) = $4,940.24 \) and \( P(-100bp) - P(0) = -$4950.86 \).

2. To prove that a given function is convex (concave) we use definitions and the second derivative test (Proposition 2.2 on p.11)

   (a) \( f(x) = x \), \( f'(x) = 1 \) and \( f''(x) = 0 \). Obviously \( f'' \geq 0 \) on \( J = (-\infty, +\infty) \), and \( f \) is convex;

   (b) \( f(x) = 1/x \); \( f' = -1/x^2 \) and \( f'' = 2/x^3 \); \( f'' > 0 \) on \( J = (0, \infty) \), so \( f \) is convex;

   (c) \( f(x) = e^x \); \( f' = f'' = e^x > 0 \) and \( f \) is convex on \( J = (-\infty, +\infty) \);

   (d) \( f(x) = \max[k, x - k] \) on \( J = (-\infty, +\infty) \). We consider two cases: \( x < 2k \) and \( x \geq 2k \). It is easy to check that in both cases \( f'' = 0 \), so \( f \) is convex;

   (e) \( f(x) = \ln(x) \); \( f' = 1/x \) and \( f'' = -1/x^2 < 0 \) on \( J = (0, \infty) \), so \( f \) is concave;

   (f) \( f(x) = \sqrt{x} \); \( f' = 1/2x^{-1/2} \) and \( f'' = -1/4x^{-3/2} < 0 \) on \( J = (0, \infty) \), so \( f \) is concave;
(g) \( f(x) = \exp(-1/2x^2) \), \( f' = -xf(x) \) and \( f'' = (x^2 - 1)f(x) \). We have that \( f'' < 0 \) on \( J = (-1, +1) \) and \( f'' \geq 0 \) on \( J = (-\infty, -1] \cup [1, +\infty) \).

3. We have a 10 year bond having a price 100. Easily we find DV01=0.07795. During the first 6 months the bond is special and the repo rate is 200bp lower than the general collateral repo rate. After that the bond is general collateral flat and the repo rate is the same as the general collateral repo rate.

Over the first 6 months the bond will have a saving in terms of funding (compared to a general collateral) that corresponds to \( S = 100 \cdot (0.02) \cdot (0.5) = 1 \). Here we used the money market convention with \( T = 1/2 \). The price of the bond has to be a point higher than the price of the bond that is not special, so we need \( 1/(0.07795) = 12.8 \) bp of variation of the yield. Therefore, the yield of a general collateral flat 10 year bond has to be 12.83bp higher than the yield of a 6 months special 10 year bond. The latter is 5% and hence we obtain \( y \approx 5.13\% \).