Solution Set 1

1. Each payment is \( x = \frac{50,000,000}{25} = 2,000,000 \). The interest rate is \( r = 7\% \). We can use the annuity formula for 24 years and add $2,000,000:

\[
PV = 2,000,000 + PV(24; 0.07; 2,000,000)
\]
\[
= 2,000,000 + \frac{2,000,000}{0.07} \left( 1 - \frac{1}{(1.07)^{24}} \right)
\]
\[
= 24,938,668
\]

2. If the growth rate is \( g \) and the earnings come at the end of each year, using growing perpetuity formula we have:

\[
PV = 50 = \frac{x(1 + g)}{r - g} = \frac{1}{0.1 - g}
\]

Solving this equation for \( g \), we obtain:

\[
g = \frac{4}{51} \approx 7.84\%
\]

Note that here \( g \) can not be greater than \( r \!\).%

3. Assume we will live 50 more years, interest rates are fixed at 5\% per annum and we plan to spend $100,000 per year. Then using the annuity formula we obtain:

\[
PV = \frac{100,000}{0.05} \left( 1 - \frac{1}{(1.05)^{50}} \right) = 1,825,593
\]

Using the growing annuity formula we can factor in inflation, etc.

4. If the first payment occurs at the beginning of the first year, we have:

\[
PV_0 = 100 + \frac{100}{(1 + r)^2} + \frac{100}{(1 + r)^4} + \cdots = 100 \sum_{k=0}^{\infty} \frac{1}{(1 + r)^{2k}} = \frac{100(1 + r)^2}{r(r + 2)}
\]
If the first payment occurs at the end of the first year, we have:

\[ PV_1 = \frac{100}{(1 + r)} + \frac{100}{(1 + r)^3} + \frac{100}{(1 + r)^5} + \cdots = PV_0 \frac{100(1 + r)}{r(r + 2)} \]

5. Simply you can divide the growing annuity formula by \(1 + g\) to obtain the answer:

\[ \frac{x}{(r - g)} \left(1 - \left(\frac{1 + g}{1 + r}\right)^n\right) \]

6. After time \( t \) a dollar compounded continuously at \( r\% \) yields \( e^{rt/100} \). Then the equation \( e^{rt/100} = 2 \) implies that the money doubles after \( 100 \ln 2 / r \) which is roughly 69.

7. See lecture notes.