INTRODUCTION

The theory and practice of financial risk management has its roots in the broader and older field of risk management in a general context. This broader field of risk management is generally termed "decision analysis" and forms a subdiscipline of statistics, operations research, economics, and psychology.

Is financial risk management just the application of these broader decision analysis principles to the specific domain of decisions faced by financial firms (i.e., banks, investment banks, and asset management firms)? In some ways this appears to be the case. Certainly we can recognize many of the standard paradigms of decision analysis in financial risk management — such as the discussion of expected value, risk aversion, and risk reduction through diversification. But financial risk management has one primary characteristic which distinguishes it from the broader field, its heavy reliance on using the market prices of various risks as a focal point.

Why is this characteristic so important? We'll go into more detail, but a short answer is that traditional decision analysis often bogs down due to the difficulties in correctly assessing the probabilities and measures of risk aversion it requires to arrive at a recommended decision. This makes it difficult for an individual to apply the theory and even harder for a group of individuals to utilize the theory to resolve different viewpoints. The great success of the theory of financial risk management has been its ability to substitute market prices which can be objectively determined for some (but not all) of the probabilities and risk aversion measures needed for decision analysis. This has allowed financial risk management to achieve far more precise results than traditional decision analysis and has opened a career path to quantitative analysts whose skills lend themselves to extracting precise consequences from such prices. So great has this success been that decision makers in non-financial fields are seeking to substitute techniques drawn from financial risk management for the traditional decision analysis techniques. This approach, termed "real options", can only be successful to the extent that market prices can be found for the appropriate type of non-financial risk (a good, brief introduction to the topic can be found in the article “Real options: let the buyer beware” by Reuer and Leiblein in the Financial Times of May 3, 2000). Which is a good reminder of why these techniques have originated and been successful in the financial industry — the dominance in this industry of open markets in a wide variety of products — what is termed in economics the presence of "complete" (or near-complete) markets.

Let us consider a categorization of financial risks which will be useful to us in further understanding the nature of financial risk management, the distinction between risk disaggregation and risk aggregation. Risk aggregation attempts to reduce risk by creating portfolios of less than completely correlated risk and so achieve risk reduction through diversification. Risk disaggregation attempts to reduce risk by breaking a risk which cannot be directly priced in the market into subcomponents, some of which can be priced in the market (for example, if there is no market for puts but there is a market for calls and for the underlying, we could disaggregate a put into a call plus a position in the underlying). Note that if markets were really complete, there would be no need for disaggregation — all prices needed would be immediately available in the market. Equally, disaggregation would not be possible without open markets in some of the components.

While much focus in the past few years has been on risk aggregation techniques such as value-at-risk and stress testing, risk reduction through disaggregation remains a vital part of financial risk management activity and also places the heaviest demands on the modeling skills of quantitative analysts. In addition, risk aggregation techniques in financial risk management also depend on
availability of market prices for risks. Without market prices, the probabilities and correlations needed for risk aggregation would rest on the same imprecision as traditional decision analysis. Lack of ability to represent the prices at which risk could, if desired, be neutralized within reasonably short time periods (e.g., several weeks) would necessitate the use of the difficult-to-achieve long-term projections required by traditional decision analysis. In this way, good risk aggregation requires good risk disaggregation, so that needed market prices can be inferred from observable ones.

Let's look in more detail at why having market prices readily available makes such a large difference in how decision analysis is performed. To keep our analysis somewhat concrete, we will focus on a particular case, how a financial firm will decide whether or not to invest in a complex instrument. We will see how the nature of analyzing this decision changed with heavier utilization of market prices in the analysis. There is at least a rough correspondence between the story we are telling here and the actual evolution of decision making within financial firms.

Let's start in the 60s, before there was significant use made of market prices. The analysis then, to the extent it was formalized at all, would have utilized classical decision analysis techniques. Different possible future scenarios would be identified, with probabilities assigned to each scenario, \( P_s \), such that \( \sum_s P_s = 1 \) for all \( s \). For each scenario, a set of cash flows \( CF_{T,S} \) would be determined which the investment would provide at future time \( T \) given scenario \( S \). Given a set if discount factors, \( D_T \), for known (i.e., risk free) cash flows at time \( T \), the value of the investment to the firm would be

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Where \( U \) is the firm's utility function, representing its degree of risk aversion.

How does classical decision theory say we are supposed to determine these probabilities, the \( P_s \)'s? The dominant theories prior to the 20th century were to either assign equal probabilities to a set of cases which were considered equally probable on intuitive grounds (for example, assigning 1/6 probability to each side of a seemingly fair die) or assigning probabilities based on long-run observations of frequencies. But the first of these two methods is obviously only applicable to a quite limited number of situations and the second of the two methods raises questions of how to determine which is the relevant series of observations to use in a particular case. Should you base your frequencies for the price movements of a particular stock on just the history of that stock or on a history of stocks within the same industry or country? Should price observations from 100 years ago be excluded as no longer relevant? The lack of clear standards for answering such questions has led most practitioners of decision analysis to favor a "personalist" or "subjective" approach to probabilities, in which probabilities are taken to represent a particular individual's "betting odds."

A major component of classical decision analysis is techniques for extracting from senior managers their betting odds and utility functions. In practice, it has generally been found that managers are quite uncomfortable with these procedures and find it difficult to pin down values which are sufficiently precise to allow decisions to be made. Exacerbating the problem is that risk aversion raises very small probabilities to great importance. A rare economic occurrence, such as a disastrous sequence of terrible weather leading to severe shortages of some commodity, which would make very little contribution to an expected value calculation, might be an event which threatens a firm with bankruptcy and so receives a significant weight from a risk-averse
utility function. But experiments have shown that almost all individuals perform extremely poorly at estimating probabilities for rare events, since the very rarity of the event implies that they seldom get feedback which allows them to correct errors.

The actual decision-making process in a large firm will often quickly degenerate into a highly politicized process. In the absence of precise guidelines for determining probabilities and utility functions, the temptation is strong (and seldom resisted) for proponents of a particular action to "tweak" probabilities and utilities until they find values which support their point. For example, a team which has been working on putting together the structure of a complex investment has strong psychological and monetary motivation to find arguments to support the desirability of going ahead with the investment. If an initial analysis shows the transaction undesirable, a little sensitivity analysis can determine how much probability assumptions and utility measures need to be altered to change this outcome. It is easy for a motivated team to convince itself of the rightness of their new assumptions. Persuasion of other decision makers may then hinge on internal political power.

The revolution in analysis which hit the financial markets in the 70s and early 80s was the replacement of much of the subjective judgment about probabilities and utilities with actual market prices at which contingent cash flows (i.e., scenario-dependent cash flows) could be purchased and sold. This revolution consisted of two mutually reinforcing trends: (1) the growth of publicly traded markets in contingent claims such as options, (2) the development of arbitrage-based analytic techniques for determining combinations of publicly traded contingent claims which could come close to reproducing complex scenario-dependent instruments. These two components were mutually reinforcing because the advances in analysis led to a greater demand for publicly traded contingent claims, as more users of complex instruments were motivated to use them for reducing uncertainty, while the availability of more publicly traded hedging instruments widened the scope of building blocks available for arbitrage analysis. While the development of arbitrage analytic techniques did not begin or end with the Black-Scholes-Merton model of 1974, this did represent a watershed event in introducing dynamic hedging with instruments which are not contingent claims as a technique for bridging the gap between payoff profiles of complex instruments and those which could be directly replicated through combinations of publicly traded instruments.

The use of publicly traded instruments to create close replications of complex instruments greatly reduced the need for subjective probability estimates and utility functions, which were now needed to only cover the remaining gaps left by imperfect replication. This significantly reduced the scope for political manipulation of results and expanded the influence of mathematically trained personnel who could provide the relatively objective arbitrage analysis which was required.

Arbitrage analysis can be represented in many differing but mutually consistent ways. One way, emphasized by the risk-neutral valuation approach is to continue to analyze the value of a complex scenario-dependent transaction by equation (1),

$$\sum_{s} \left( \sum_{t} CF_{t,s} D_{T} P_{S} \right)$$

but to utilize for the $P_{S}$'s probabilities which reproduce the values of publicly traded contingent claims. Note that the utility function $U$ has now been dropped, since the use of replication has to some degree of approximation eliminated uncertainty. As is often emphasized, these are not "true" probabilities in the sense of representing anybody's views of the future, not even an average view of market participants. Rather they are pseudo-probabilities, i.e., weights which
add up to 1 and which reproduce values which can be achieved by hedging in the publicly traded market. As such, the argument for using these values as probabilities is only as strong as the extent that hedging with these publicly traded instruments is truly an available alternative.

Of course, it is only an approximation to view instruments as being publicly traded or not. The amount of instruments available for trading differs widely by size and readiness of availability. This constitutes the "depth" of "liquidity" of a given market. Often a firm will be faced with the choice between the risks of replicating positions more exactly with less liquid instruments or less exactly with more liquid instruments.

A dominant theme of this course will be the trade-off between liquidity risk and basis risk. Liquidity risk is the risk that the price at which you buy (or sell) something may be significantly higher (lower) for you than the price you could have gotten under more ideal conditions. Basis risk is the risk that occurs when you buy one product and sell another closely related one and the two prices behave differently. Let’s take an example. Suppose you need to sell a house If you need to sell it very fast, let’s say within a week or two (maybe because you’re in desperate financial straits), you might expect that you would not get as good a price as you could if you had a longer period of time in which to wait for a higher offer. This is liquidity risk based on time. You could also have liquidity risk based on geography. If you were limited to selling your house only to people live in your small village, you might not obtain as good a price as if you had the services of a real estate agent who would aggressively market your house to people outside your town. Basis risk in this context, would come about if in the course of keeping your house on the market over a long period of time, in order to lessen liquidity risk, you fall victim to a slump in the economy which decreased the price of all houses. This is a very typical situation in that there is a trade-off between liquidity risk and basis risk. The sooner you sell the house after buying it, the less basis risk there is but the greater the degree of liquidity risk, and vice versa.

Frequently the only way in which liquidity risk can be reduced is to increase basis risk and the only way in which basis risk can be reduced is to increase liquidity risk.

Illustration: The design of a set of futures contracts.

Narrowing the definition of each individual futures contract — and therefore increasing the number of different futures contracts — will reduce the basis risk to a user of the contract. If each contract specifies a narrow range of delivery grades (e.g., quality level of a commodity, coupon and maturity date of a bond), place of delivery (important for commodities due to the cost of physical transportation), and time of delivery, basis risk for a contract user will be small. But liquidity risk will be large for two reasons: (1) infrequency of trade in any one contract means that fairly small bids and offers will move the market, (2) it may be difficult for a user who is contracted to deliver to find an exact match — he/she may be forced to pay an exorbitant price — leads to attempts to “corner” the market to “squeeze the shorts.”

Liquidity risk can be reduced by broadening the definition of each individual futures contract — decreasing the number of different futures contracts — to include a range of delivery grades, places of delivery, and times of delivery. But this will increase the basis risk from pricing mismatch between the position to be hedged and the position delivered. For the futures buyer, the basis risk comes from not knowing exactly what will be delivered. The futures seller can control the content of what is delivered but still has basis risk from the fact that futures prices may be driven by price movements in grades, delivery places, and delivery times which differ from the combinations he/she will deliver.
Another key distinction we will be emphasizing in many different contexts throughout this course is that between static hedging and dynamic hedging. In risk disaggregation, we attempt to break a risk which cannot be priced directly in the market into subcomponents which can be more readily priced in the market. Our preference will always be a disaggregation using static hedging, one in which the subcomponents you use today will be exactly the same subcomponents which are equivalent to the instrument being disaggregated throughout its life. When you price an instrument by disaggregation using static hedging, there is no residual uncertainty — you could if you wish go out and purchase these hedges and not take any other actions.

But static hedges are not always available. When they are not, we will need to use dynamic hedging in which the disaggregation into subcomponents changes over time. But future shifts in hedges will likely have costs which cannot be known today, leaving a residual uncertainty around any pricing using the risk disaggregation. The range of uncertainty will, of course, vary from product to product. A major advantage of the Bachelier-Scholes-Merton theory for pricing European options is that it provides a formula for estimating the cost of a dynamic hedge. In fact, if the simplifying assumptions used to derive the formula were really true in practice (primarily that hedging can be done as frequently as wished with no transaction costs, that there are no price jumps, and that volatility is a known constant) then the future cost of dynamic hedging would be known exactly. In practice, these assumptions are not strictly true, but in many markets they are close enough approximations to the truth that they can be used to reduce the uncertainty of dynamic hedging costs to within a manageable range. How this can be done will be a major topic of this course.

Some dynamic hedges require fewer future changes in hedges than others and may be called quasi-static hedges. The extreme cases are dynamic hedging strategies which call for at most one change in the hedge over the life of the instrument. Quasi-static hedges simplify the task of estimating uncertainty of future hedging costs by allowing us to focus on only a few, maybe only a single, point in time. In this course, we will study several quasi-static hedging techniques, including the “stack and roll” techniques for hedging long-dated forwards and long-dated European options and several possible quasi-static hedges for barrier options.

**What is risk management as applied to trading?**

When applied to trading, risk management has three distinct though related meanings. First, risk management means that area of the bank which creates and trades in financial instruments which individuals and corporations use to manage their financial risks. So, for example, the risk management area would be the one which trades in foreign exchange and creates interest rate derivatives for use by individuals and corporations. Second, risk management refers to the activity which any trading desk engages in to mitigate and control the risks of trading. In this sense of the term, risk management is often used as a synonym for portfolio management as opposed to structuring and marketing. A synonym for the head risk manager could be either the head portfolio manager or the head trader. The contrast is that marketers/structuring are only concerned with the transaction until it reaches its final agreement with the other party to the transaction whereas risk managers must continue to be concerned with the consequences of the transaction until its final termination. The third meaning of risk management refers to the specialized staff functions within the firm which deal with particular aspects of risk. These include managers of market risk, credit risk, legal risk, accounting risk, operations risk, and liquidity risk.
In this course, we will be focusing on both the management of risk by trading desks and on the firm-wide management of risk by specialist groups. We will only tangentially be concerned with how instruments created by financial firms are utilized to manage the risk of non-financial firms and individuals though the considerations of internal risk management will have a strong influence how these risk management products are priced to customers.

There are two fundamental, and complementary, approaches to the management of risk. The first is for higher levels of management to place limits on the amount of risk which lower levels of management can take. The second is for higher levels of management to provide incentives to lower levels of management to optimize the tradeoff between return and risk. Each approach has advantages, pointing towards a use of both approaches. The incentive approach gives lower levels of management, which are closer to the information required to make tradeoff decisions, the flexibility to find combinations of risks which can maximize return for a given risk level. But the incentive approach, by focusing on micro-level decision making, can lead to unacceptable concentrations of certain risks when aggregated up to the firm level. Such risk concentrations are better controlled through the limits approach. The incentive approach, by its broad-brush nature, is also more vulnerable to “gaming” by lower levels of management, and the limits approach is needed as a check on this possibility.

The key elements in the limits approach are measurement of maximum potential loss due to different risk factors (e.g., interest rate movements for market risk, the default of a particular borrower for credit risk, incorrect trade entries for operational risk). One key difference between market risk on the one hand and credit and operational risk on the other is that credit and operational risks are always undesirable and much of the effort in limiting these risks in devoted to finding methods to reduce these risks as low as possible, while market risk represents both a potential for profit and a potential for loss. To the extent a firm believes it has knowledge about the direction of interest rates it has a positive desire to take on interest rate market risk and would avoid possible measures to mitigate this exposure. Risks need to be measured for limit purposes at both overall portfolio levels (e.g., total credit risk exposure) and at sublevels (e.g., total credit exposure to a particular counterparty or particular industry) because of the goal of preventing overconcentration.

Key elements in the incentive approach are correct measurement of return and measurement of portfolio risk. Market risk principles for correct mark-to-market valuation procedures, model reviews, and valuation reserves, and credit risk adjustment of return for expected credit losses are examples of correct return measurement. The measurement of portfolio risk must include elements of the firm’s internal assessment of risk and regulatory, financial analyst, and rating agency assessment of risk, which may impose costs on the firm in the form of required capital. Return and risk are combined through a measure such as RAROC (risk adjusted return on capital) or SVA (shareholder value added).