Fifth Assignment (Due Wednesday March 3rd)

The dispersion relation for linear one-dimensional surface waves over water of mean height $H$ is given by

$$\omega = \sqrt{g |\xi| \tanh(H |\xi|)},$$

where $g = 10 \text{ m/s}^2$ is the gravity constant (In fact, the same dispersion relation applies to two-dimensional waves, but we shall concentrate here on the one-dimensional case for simplicity.) The effects of a stone dropped into a river can be modeled by the initial value problem for $\eta(x,t)$, the height perturbation, given by

$$\eta(x,0) = \frac{d}{2} e^{-\left(\frac{x}{d}\right)^2}$$

where $d$ is the diameter of the stone. With a mean height $H$ of two meters, solve this problem and plot the solution at times $t = 0$, $t = 2$ and $t = 10$ seconds, for stones of diameter equal to twenty centimeters, fifty centimeters and two meters. Compare your solutions qualitatively (do these results agree with your experience of dropping stones into rivers?), and also discuss them quantitatively in terms of the group velocity.

This problem is to be solved analytically, but you may find the little matlab program Dispersion.m, in the class webpage, to be of use for checking your results.

Note: For stones smaller than about 10 cm in diameter, the effects of surface tension can no longer be neglected; this accounts for a different dispersion relation, and quite different phenomena.