Homework #3 • Applied Math G63.2701 • Numerical ODEs – Runge-Kutta

- due monday 12 october – in lecture.
- as always: correctness, clarity & conciseness.
- include matlab plots & results in your write-up.
- annotate plots & point out key features, include relevant discussion on the plot.
- there is no need to submit matlab scripts.

*) Download from the class web-page (look for me under math.ryu.edu) the matlab scripts for the sample Runge-Kutta code. The code should be set-up to solve the simple 4th-order ODE

\[ \ddot{\mathbf{Z}} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \mathbf{M} \ddot{\mathbf{Z}} \]

with \( z_1(0) = z_2(0) = z_3(0) = z_4(0) = 1 \) as initial conditions. Running the script rk_{ex} should produce some rudimentary plots of the solution from \( 0 \leq t \leq 0.5 \) with \( \Delta t = 0.005 \) as the timestep (100 steps).

This script represents only the most primitive of templates for a Matlab tool for ODE study. You should make several copies and modify them to suit your needs.

For two exactly-solvable ODEs of your choice – A) a 1st- and B) a 2nd-order ODE – present Matlab computations that illustrate some interesting aspect of ODE solutions. Remember the list of possibilities:

- non-constant coefficient,
- non-homogeneous forcing,
- nonlinearity,
- varying initial values, or
- varying parameters.

Finally, for both ODEs include an error analysis of a single trajectory. That is, compare the accuracy of your numerical solution with the known, exact solution – as it is affected by the choice of timestep size (\( \Delta t \)). Make a plot of

\[ \log_{10} |\hat{y}(T) - y(T)| \quad \text{versus} \quad \log_{10} \Delta t \]

where \( \hat{y}(T) \) and \( y(T) \) are the Runge-Kutta and exact solutions at the final time \( t = T \). This MUST provide quantitative verification that your computations are 4th-order accurate (why?). Investigate timesteps over a wide range of \( \Delta t \). Explain the eventual breakdown of the error analysis in the limits of small and large timesteps. For what \( \Delta t \) would you consider your computations are reliable? Your timestep intervals need not be finely spaced (8-10 runs for well-dispersed \( \log_{10} \Delta t \) should illustrate the point); use discrete markers.

You should submit only 3-5 pages per ODE including plots! Condense essential information onto fewer plots – the subplot feature might be convenient.