A) **The Hollow Planet:** Consider a universe containing a single, lonely planet. This planet consists only of a thick, but hollow crust having uniform mass density. Assuming a spherically-symmetric geometry, solve for the gravitational potential $V(r)$ and the magnitude of the force of gravity $F(r)$

$$\nabla^2 V = 4\pi G \begin{cases} \rho_0 & \text{for } 0 < R_1 < r < R_2 < \infty \\ 0 & \text{otherwise.} \end{cases}$$

The solution is determined by applying regularity conditions at the origin, and continuity (of both potential and force) at each of the planet’s inner and outer surfaces ($r = R_1, R_2$). Note that these conditions completely specify the behavior of the force at infinity – verify that this is the expected result using a 3D Greens Theorem. Make a matlab plot of a representative solution, and state conclusions.

B) **Poisson in the Square:** On a unit square $0 \leq x, y \leq 1$, derive an integral representation for the Poisson problem

$$\nabla^2 u = H(x, y) \quad \text{with the BC} \quad U(\text{bndry}) = 0.$$

By approximating the sum numerically in matlab, you should find that the kernel function $K(x, y; \bar{x}, \bar{y})$ is singular. Show numerically that the kernel has the form

$$K(x, y; \bar{x}, \bar{y}) = C \ln r + \tilde{K}(x, y; \bar{x}, \bar{y})$$

where $r^2 = (x - \bar{x})^2 + (y - \bar{y})^2$ and $\tilde{K}(\bullet)$ is non-singular. What is the constant $C$ and is it independent of $\bar{x}, \bar{y}$? Plot the non-singular part $\tilde{K}(x, y)$ for several values of $\bar{x}, \bar{y}$.

C) **Laplace in the Circle:** Give the solution for the Laplace equation in the circle for non-homogeneous Neumann boundary condition, $u_r(a, \theta) = g(\theta)$. State the fine print regarding the existence and uniqueness of the solution.

D) **Laplace in the Half-Plane:** GL, p305 #5 and the limit in #6b. Consider the change of variable $t = (x - s)/y$. 