These problems provide practice with normal and lognormal distributions and their consequences. (For more information about the actual distribution of market returns, one good source is: J. Case, “The modeling and analysis of financial time series,” *American Mathematical Monthly* 105 (May 1998) pp. 401-411.)

(1) [Ross, exercise 2.6] Suppose the amount of time a certain battery functions is a Gaussian random variable with mean 400 hours and standard deviation 50 hours. Consider an individual who owns two such batteries, one of which is to be used as a spare to replace the other when it fails.

(a) What is the probability that the total life of the batteries will exceed 760 hours?

(b) What is the probability that the second battery will outlive the first by at least 25 hours?

(c) What is the probability that the longer-lasting battery will outlive the other by at least 25 hours?

(2) [Ross, exercise 2.7] Suppose the time it takes to develop a photographic print is a Gaussian random variable with mean 17 seconds and standard deviation 1 second. Estimate the probability that the total time it takes to develop 100 prints is

(a) more than 1710 seconds;

(b) between 1690 and 1710 seconds.

(3) [Ross, exercise 2.10] Consider the following “additive trinomial tree” model for the dynamics of a stock: if the price at period $n$ is $s_n$, then price at period $n+1$ is either $s_n - 1$ (probability .39), $s_n$ (probability .20), or $s_n + 1$ (probability .41). Assume distinct periods are independent. Estimate the probability that after 700 periods the price will have risen at least 10, i.e. the probability that $s_{700} - s_0 \geq 10$.

(4) [Ross, exercise 2.9] Consider the following “multiplicative binomial tree” model for the dynamics of a stock: in each period the price either goes up by a factor of $u = 1.012$ or down by a factor of $d = .990$. Assume the probability of going up is $p = .52$, and distinct periods are independent. Estimate the probability that the stock’s price will be up at least 30 percent at the end of 1000 time periods.

(5) Suppose the annual return on a certain investment is lognormal with mean 10% and standard deviation 20% (by which I mean $R_{ann} = e^X$, where $X$ is Gaussian with mean .10 and standard deviation .20). Find

(a) The likelihood of the return being at least 5% in any given year.
(b) The likelihood that value of the investment 10 years from now is at least 50% greater than it is now.

(c) The likelihood that the value of the investment 10 years from now is at least \((1.05)^{10}\) times its present value.

(d) The likelihood that the value of the investment increases by at least 5% in each of the next 10 years.

(6) Suppose the annual return on a certain investment is lognormal with mean 5% and standard deviation 15%.

(a) What is the likelihood of a loss over the coming year (i.e. the likelihood that the value a year from now will be less than it is today)?

(b) What is the likelihood of a loss over at least 1 of the next 5 years?

(c) What is the likelihood of a loss over at least 2 of the next 5 years?

(7) Suppose \(R\) is lognormal with mean \(\mu\) and standard deviation \(\sigma\), i.e. \(R = e^X\) where \(X\) is normal with mean \(\mu\) and standard deviation \(\sigma\).

(a) Show that the mean of \(R\) is \(e^{\mu + \frac{1}{2} \sigma^2}\). [Hint: see problem 4 in chapter 11 of Luenberger for an outline of the calculation.]

(b) Find the variance of \(R\). [Hint: All the hard work has already been done in part (a). Use the formula \(\text{Var}(R) = E[R^2] - (E[R])^2\), and notice that if \(R\) is lognormal then so is \(R^2\).]