Recall notation + basics of large pool, kroup, base correlation, single-factor, Gaussian copula model:

- Focus on assessing losses due to default up to a fixed time \( t \). Assume \( D = D(t) = \text{prob that any single name has defaulted by time } t \) is known. Also, \( \rho = \text{correlation is known (constant + uniform)} \). Then by 1-factor hypothesis + Gaussian copula model, with \( M = \text{the common factor} \),

\[
\text{prob of default of } i^{th} \text{ name by time } t, \text{ given value of } M \text{ is}
\]
\[
N\left( \frac{N(D) - \frac{\sqrt{\rho}}{\sqrt{1-\rho}} M}{\sqrt{1-\rho}} \right) = \Omega_t(M)
\]

- Under "large pool" hypothesis, we use law of large numbers to say \( \Omega_t \) is the "exact" fraction of pool that has defaulted by time \( t \).

What to do with this? Well, for any single tranche of a CDO that's based on
CDS's (or "synthetic CDS") there are two cash flows:

\[ N \frac{\delta}{f} \cdot \frac{q(S_t^+)}{j} \text{ at the } t_f \]

where \( \delta \) is spread, \( N \) = principal, \( f \) = freq., and \( q(S_t^+) \) is a piecewise-linear function reflecting the attachment + detachment pts of tranche under consideration, e.g. 3% to 7% tranche.

\[ q(S_t^+) \text{ fraction of total } \]

\[ 4\% \quad \frac{4\%}{3\%} \quad 7\% \]

\[ 3\% \quad 4\% \quad 5\% \]

Value the payment by taking expected value + roll by discount factor

\[ N \frac{\delta}{f} B(0, t_f) \cdot E[ q(S_t^+ \mid H) ] \]

where \( E \) = expectation w.r.t. Gaussian statistics of \( H \). Calculate expectation numerically, e.g. by Gaussian quadrature (or any numerical integral scheme).
\[ N(1-R) \left[ -g(S^*_t) + g(S^*_{t-1}) \right] \]

where recovery rate is \( R \), and

\[ -g(S^*_t) + g(S^*_{t-1}) = \text{fraction of total prin.} \]

\[ \text{assoc. this tranche that defaulted during most recent period} \]

This is calculated numerically, since it is linear, we need only calculate

\[ E \left[ g(S^*_t(N)) \right] \]

at \( t + t_1 \). (Calculations differ only by having different values of \( D = D(t) \).)

Discount of course to get present value

\[ N(1-R) B(0, t_1) \left[ E \left[ g(S^*_t(N)) \right] - g(S^*_{t}) \right] \]

Stevens table (p. 55) demonstrates calc of

\[ E \left[ g(S^*_t(N)) \right] \]

using crude Gaussain quadrature (4 quad. pts., so it's possible to do by hand)
for two choices of tranche

1. The 0% to 3% base tranche
2. The 0% to 7% base tranche

Note that for base tranches $q$ looks like

\[ q^{7\%} \quad \frac{g}{g^{7\%}} \quad 0 \quad 5\% \]

\[ q^{7\%} = g^{7\%} \quad 0 \quad 7\% \]

So

\[ q^{7\%} - g^{5\%} \]

is precisely the $g$ associated to the 5% to 7% tranche. Thus for computing expected losses we may always use base tranches then subtract. (Base tranches are more convenient for defining an implied correlation, since for base tranches the expected loss is monotone in $q$.)
Suggestion for further reading that's consistent with viewpoint discussed here: