Assignment 5.

Given November 15, due November 29.

**Objective:** Simulate a stochastic process

We want to include default risk in pricing a simple zero coupon corporate bond. In this model, we say that the company defaults if its stock price ever goes below $S_d = 1$. Let $D$ be the event that $S(t) \leq S_d$ for some $t \leq T$, and let $ND$ be the complementary event. The random variable $\chi$ will have the value 1 if $ND$ and 0 if $D$. The risk adjusted present value is

$$E \left[ \exp \left( - \int_0^T r(t) dt \right) \cdot \chi \right].$$

Use the following model:

$$dS = rS dt + \sigma_S S dW_1$$

$$dr = \mu (r_0 - r) dt + \sigma_r r dW_2$$

$$d\sigma_S = \nu (\sigma_0 - \sigma_S) dt + \sigma_v \sigma_S dW_3$$

with parameters

$$r_0 = 7\%$$

$$r(0) = 6\%$$

$$\sigma_S(0) = 50\%$$

$$\mu = .5$$

$$\sigma_r = 30\%$$

$$\nu = 2$$

$$\sigma_v = 80\%$$

$$\sigma_0 = 30\%$$

$$S(0) = 10$$

$$T = 10$$

Assume the correlation of $W_1$ and $W_2$ is -.3, the correlation of $W_1$ and $W_3$ is -.5, and the correlation of $W_2$ and $W_3$ is .4.

Use the forward Euler method to generate sample paths and then average the functional. Do an experiment with several fairly large values of $\Delta t$ to see how the bias depends on $\Delta t$. 