Assignment 6.

Given March 23, due April 13.

Objective: To explore numerical methods of ODE’s.

We want to compute the trajectory of a comet. In nondimensionalized variables, the equations of motion are given by the inverse square law:

\[
\frac{d^2}{dt^2} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{-1}{(x^2 + y^2)^{3/2}} \begin{pmatrix} x \\ y \end{pmatrix}.
\]

We will always suppose that the comet starts at \( t = 0 \) with \( x = 10, y = 0, \dot{x} = 0, \) and \( \dot{y} = v_0 \). If \( v_0 \) is less than the escape velocity (\( 1/\sqrt{5} \) here), the comet will move in an elliptical orbit, returning to \( x = 10, y = 0 \) after a time \( T(v_0) \). This is the time for a single orbit.

1. Compute the position and velocity at time \( t = 30 \) with \( v_0 = .2 \) using a fixed \( \Delta t \) and the three methods: Forward Euler, second order Adams Bashforth, and four stage fourth order Runge Kutta. Do a convergence study to verify the expected order of accuracy of each of these methods.

2. Use a fixed time step to compute \( T(v_0) \) as a function of \( v_0 \) for \( v_0 \) in the range \([.01, .25]\). Plot the result. For this, you will need some way to pick an appropriate time step, possibly by doing a convergence study. You will also have to safeguard your program against the possibility that \( T(v_0) \) is infinite\(^1\).

3. Write an adaptive time step program that uses the fourth order Runge Kutta together with a local truncation error strategy such as the one discussed in class. The adaptive program should much more efficient than the fixed step method when \( v_0 \) is small.

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\(^1\)“This is not a drill.”