Given: November 10
Due: November 15

1. Section 7.1, #19, 20, 21.
2. Section 7.3, #1, 2, 8, 9, 15, 16, 24.
3. Section 7.5, #5 (by hand without computer), 11.
4. Consider the second order differential equation for a two component column vector
   \[ \ddot{x} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \dot{x} + \begin{pmatrix} 0 & -4 \\ 2 & -4 \end{pmatrix} x. \tag{1} \]
   An exponential solution takes the form \( x(t) = e^{rt} \xi \), where \( \xi \) is a two component column vector.
   (a) For a general equation \( \ddot{x} = A \dot{x} + Bx \), write the equation in terms of matrices \( A \) and \( B \) and the vector \( \xi \) and the number \( r \) that we have to solve to find exponential solutions. What is the matrix that has to be singular in order for there to be \( \xi \neq 0 \)? Hint: it involves matrices multiplied by \( r \) and \( r^2 \), one of them being the identity matrix.
   (b) Write this matrix for the specific problem (1).
   (c) Calculate the determinant of this matrix, which is a polynomial of degree 4 in \( r \).
   (d) Show that this polynomial factors into a product of quadratics one of which is \( r^2 + r + 4 \). Find the other factor.
   (e) List all the numbers, \( r_1, r_2, r_3, \) and \( r_4 \), that correspond to exponential solutions of (1).
   (f) What kind of behavior do they represent? (growth/decay, simple/oscillatory)
   (g) Find the \( \xi \) corresponding to \( r = 1 + i \).
   (h) Take the real part of \( e^{rt} \xi \) from part g to find a real solution of (1).
5. In each case there are three elements of the vector space of functions of \( t \). Either show that the functions (vectors) are linearly independent or find a linear combination \( 0 = c_1 f_1(t) + c_2 f_2(t) + c_3 f_3(t) \).
   (a)
   \[
   \begin{align*}
   f_1(t) &= t(t - 1) \\
   f_2(t) &= (t - 2)(t - 3) \\
   f_3(t) &= (t + 2)(t + 3)
   \end{align*}
   \]
(b)

\[ f_1(t) = \sin(t) \]
\[ f_2(t) = \sin(2t) \]
\[ f_3(t) = \sin(3t) \]