Given: November 18
Due: November 22

1. For an \( n \times n \) matrix \( A \), the \textit{transfer function}, also called \textit{resolvent}, is \( R(s) = (sI - A)^{-1} \). Show that \( R(s) \) is defined for all complex numbers that are not eigenvalues of \( A \).

2. Show that we may find a particular solution of \( \dot{x} = Ax + ve^t \) of the form \( x(t) = we^t \) using the resolvent provided \( s \) is not an eigenvalue of \( A \).

3. Compute \( R(2) \) for the matrix 
\[
A = \begin{pmatrix}
2 & 1 & -1 \\
1 & 2 & -1 \\
1 & 2 & 2
\end{pmatrix}.
\]
Use it to find a particular solution to the equation \( \dot{x} = Ax + ve^{2t} \) where \( v = (1, 0, -1)^t \).

4. For a \( 2 \times 2 \) matrix 
\[
M = \begin{pmatrix}
a & b \\
c & d
\end{pmatrix},
\]
verify the formula 
\[
M^{-1} = \frac{1}{\det(M)} \begin{pmatrix}
d & -b \\
-c & a
\end{pmatrix}
\]
by checking that the product of the two matrices is \( I \).

5. Use the formula from question 4 to calculate \( R(s) \) where
\[
A = \begin{pmatrix}
-1 & 1 \\
-4 & 0
\end{pmatrix}.
\]

6. Use your answer to question 5, and of course complex exponentials, to find a particular solution to
\[
\dot{x} = Ax + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cos(t).
\]

7. Section 7.6, \# 4, 5 (Take real or imaginary parts of complex solutions.), 9, 26.

8. Section 7.7, \# 3, 11.

9. Section 7.5, \# 1, 11, 13.