Assignment 8, Problem 3: a bond math spreadsheet

This assignment involves creating a spreadsheet that implements some of the bond math in Section 9 of the notes by Bob Kohn and Steve Allen. The first few rows should look like this:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maturity</td>
<td></td>
<td>Price</td>
<td>Coupon</td>
<td>Early Coupon Value</td>
<td>Stripped Price</td>
<td>Final Payment</td>
<td>B(0,t)</td>
<td>Y(0,t)</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>1</td>
<td>0.98</td>
<td>0.00%</td>
<td>0.0000</td>
<td>0.9800</td>
<td>1</td>
<td>0.9800</td>
<td>4.04%</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.07</td>
<td>5.00%</td>
<td>0.2450</td>
<td>0.98250</td>
<td>1.025</td>
<td>0.9585</td>
<td>4.23%</td>
<td>0.97810</td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
<td>1.11</td>
<td>5.00%</td>
<td>0.04846</td>
<td>0.96254</td>
<td>1.025</td>
<td>0.9391</td>
<td>4.19%</td>
<td>0.97968</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1.01</td>
<td>4.00%</td>
<td>0.08223</td>
<td>0.94367</td>
<td>1.025</td>
<td>0.9201</td>
<td>4.13%</td>
<td>0.97978</td>
</tr>
</tbody>
</table>

The various columns are as follows:

**Maturity**: the time to maturity of the bond. For this spreadsheet make bonds that mature every six months, so that the maturities are 0.5, 1, 1.5, 2., etc.

**Price**: the market price of the bond “today” (at time $t = 0$). These are entered by hand.

**Coupon**: the annual coupon of the bond, to be entered by hand. The rest of the spreadsheet assumes that there are two coupon payments per year, each of half the size given in column C.

**Early Coupon Value**: The total present value of all coupon payments except the last one. For example, the number 0.049675 in cell D4 is the sum of two coupon payments of 0.025 each prices at G2*.025 (for the payment at 6 months) and G3*.025 (for the payment at one year).

**Stripped Price**: The price of the final payment, which is the total price from column B with the market prices of all coupons but the last one removed. For example, the value in E5 is the value in B5 less D5.

**Final Payment**: The last payment includes the last coupon plus repayment of the principal, which is assumed to be 1. For example, the number in F5 is a coupon payment of 0.05/2 plus the principal payment.

$B(0,t)$: The market price of a zero coupon bond maturing at $t$. Find it using the stripped price and the size of the final payment.

$Y(0,t)$: The yield (as in *yield curve*) corresponding to the value of $B(0,t)$ for the same time period.
6 month forward: Enter today into a contract give \( F_0(t,T) \) at time \( t \) and get 1 at time \( T \). Here, \( t \) is the time corresponding to the row you’re on and \( T \) corresponds to the row below, which is six months later. For example, 0.97968 in cell I6 corresponds to paying \( F \) after one year and getting 1 six months later.

Forward Yield: The implied interest rate (continuously compounded) corresponding to the forward six month forward contract just computed.

You should put the appropriate formulas into the spreadsheet as Excel formulas. Most of them you can enter once and drag to get a full table. I found that I had to edit the formulas in column D by hand because the range of the summation changes. For example, if I copy the formula \( \text{SUM(G4:G6)} \) (not the whole formula) from cell D7 to cell D8, it becomes \( \text{SUM(G5:G7)} \) rather than \( \text{SUM(G4:G7)} \). Some of the row 4 formulas are special, too.

Put in the values from the table below and all of the rest of the spreadsheet should fill in.

![Table Image]

**Action items:**

a) Make a plot of the yield curve based on the data provided. What is the shape of the curve? What expectations about future interest rates does it express? Are these views also expressed by the forward yield numbers in column J?

b) What is the arbitrage price today of a swap agreement that agrees to swap one payment at rate \( B(2.5,3) \) at time 3 (payments made at the end of the period using the rate applicable at the beginning of the period) for a fixed payment of 2.25% at time 3 (2.25 for half a year corresponds to 4.5% per annum.)?

c) What two numbers should be multiplied (give cell addresses and descriptions) to find the forward price that corresponds to borrowing at time \( t=2 \) and repaying at time \( t=3 \)? Explain your reasoning in the form of an arbitrage using two six month forwards.

d) Hand in a printout of the complete spreadsheet.