Assignment 5.

Given December 8, due end of finals week.

Objective: To work with advanced Monte Carlo methods

The portfolio is the same as that of Assignment 4. Let $V$ be the value of the portfolio at six months, as before. We want the following quantities:

\begin{align*}
\bar{v} &= \mathbb{E}(V) \\
\sigma^2_V &= \mathbb{E}((V - \bar{v})^2) \\
v_p &= \text{so that } \Pr(V < v_p) = p.
\end{align*}

We can estimate $v_p$ using “order statistics”\(^1\). Suppose $V_1, \ldots, V_N$ are the $N$ sample values of $V$ resulting from $N$ independent paths. The notation $V(k)$ refers to the same $N$ numbers, but in increasing order: $V(1) \leq V(2) \leq \cdots$. Now choose $k$ so that $k/N \approx p$, and use as estimator

\[ \hat{v}_p = V(Np) \] (4)

If $f(v)$ is the probability density function for $V$, the variance of $\hat{v}_p$ is approximately

\[ \frac{p(1-p)}{Nf(v_p)}. \]

Try to verify this computationally.

Use the SDE solver from Assignment 4 to estimate $\bar{v}$, $\sigma^2_V$, and $v_p$ with $p = 1\%$. Then redo this using as control variate the same portfolio but with European style options, for which the expected value can be found analytically. Warning: this is not the Black-Scholes “rational” price. This does not apply to $v_p$. Finally, estimate the perturbations of these three quantities when

(i) the growth rate of stock 1 goes to .35.

or

(ii) the covariance of $X_1(1)$ and $X_2(1)$ goes from .84 to .9.

Use the “different paths” method, the “same paths” method, and the “score function” method. Comment on the results. Warning: this is a class in computing, so you should comment on computational advantages and disadvantages, not on the financial significance of the answers.

When you have finished all this, try to determine a suitable way to relax and recover from a tough semester! You’re done.

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\(^1\)In statistics, the term “order statistic” refers to a statistic that depends on the samples but not on the order in which they are obtained.