Calculus I, Sections 4, 5, 6.

Practice for Quiz 5

This sample is longer than the actual quiz will be. Many of the quiz questions will be similar to these.

New rules for quiz 5 and the final: We will start taking off points for incorrect answers. If you do not know how to answer a question and leave it blank, you will get two out of ten or one out of five points. If you give an incorrect answer, we will take off up to two points to get a total as low as zero. If you check an incorrect answer and find that the answer is wrong, you may get a point for stating that the answer is wrong beyond that partial credit you would otherwise have got. Cross out all work that you know is wrong. We will deduct points for any wrong answers, even if there is a correct answer also.

1. Calculate
   
   (a) \( \frac{d}{dt} \ln(t^2) \).

   (b) \( \frac{d^2}{dx^2} e^{3x} \).

   (c) \( \frac{d}{dx} (x \ln(x) - x) \).

   (d) \( \frac{d}{dt} \left[ (e^t)^2 \right] \). Hint: Simplify the expression using properties of the exponential before differentiating.

   (e) \( \frac{d}{dx} \ln \left( \frac{x^2}{x + 2} \right) \). Hint: Simplify the expression using properties of the log before differentiating.

2. The formula \( a = e^{\ln(a)} \) expresses the fact that the base \( e \) exponential is the inverse function to the natural log. The formula is used to change base in exponentiation. For example, \( 10^3 = (e^{\ln(10)})^3 = e^{3\ln(10)} \). For changing base in log taking, we have the formula \( \ln(x) = \ln(b) \log_b(x) \). Use these formulas to calculate the following.

   (a) \( \frac{d}{dt} 2^t \). Hint: express \( 2^t \) in terms of the base \( e \) exponential then use the chain rule to evaluate the derivative.

   (b) \( \frac{d}{dx} \log_{10}(x) \).

   (c) \( \frac{d}{dt} t^4 \).
3. Calculate:

(a) $\int_3^1 \frac{1}{t} \, dt$.

(b) $\int_0^{1/2} e^{-2x} \, dx$.

(c) $\int_0^3 xe^{x^2} \, dx$.

(d) $\int_1^e \ln(t) \, dt$. Hint: look at question 1c.

(e) $\int_1^2 \frac{x^2}{x + 2} \, dx$.

(f) $\int_1^e \sin(\pi \ln(t)) \frac{dt}{t}$.

4. Find all functions of the form $x(t) = e^{kt}$ that have $\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 0$.

5. I invest $100 and after five years the investment is worth $135. The effective annual rate of return on an investment is the constant $r$ so that the value at time $t$ is given by $Ce^{rt}$. What is the effective rate of return on my investment in %/year?

6. The half life of a radioactive substance is the amount of time it takes for the radioactivity to decrease by a factor of two. A certain substance is found to lose 5\% of its radioactivity per year. What is it’s half life? How long will it take for the radioactivity to decay by a factor of 1000?