Calculus I, Sections 4, 5, 6.

Practice for Quiz 2

If you know your stuff, this should take about an hour. The actual quiz will be about a quarter this long and should take about fifteen minutes.

1. Calculate the derivatives of the following functions:
   a. \( f(x) = \frac{x}{\sqrt{1 + x^2}} \)
   b. \( f(x) = \sin\left(\frac{1}{x}\right) \)
   c. \( g(s) = \cos^2(\tan(s)) \)
   d. \( u(t) = \frac{1}{2 + \cos(t)} \)
   e. \( f(t) = \sqrt{t \tan(t)} \).

2. Compute \( \frac{d^2}{dt^2} \cos(t^2) \).

3. Suppose \( u = \frac{1}{A^2} \) and when \( t = 1 \), \( A(t) = 4 \) and \( A'(t) = -5 \). What is the value of \( \frac{du}{dt} \) when \( t = 2 \)?

4. Show that \( \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) \) exists but that \( \lim_{x \to 0} \frac{d}{dx} \left( x \sin\left(\frac{1}{x}\right) \right) \) does not.

5. Starting from a spot in Central Park, I walked 40 feet east then 30 feet north. I wind up 50 feet from the starting point. About how much farther north should I walk to increase the distance from my starting point by 2%?

6. A point \( P \) is on the circle of radius 3 centered at the origin. The point makes an angle \( \theta \) with the \( x \) axis. The point \( Q \) has coordinates \((1, 2)\). The distance between \( P \) and \( Q \) is \( F \). Write a formula for \( \frac{dF}{d\theta} \).