1. **Mortgage Basics:**

   If you take out a standard 30-year fixed-rate mortgage with interest rate \( R \% \) (compounded monthly), then you receive the original balance of \( \$B \) at time 0, and you make equal monthly payments of \( \$A \) at times 1, 2, ..., 360.

   a) Derive a simple formula expressing \( \$A \) in terms of \( \$B \) and \( R \). (Hint: Recall that perpetual payments \( \$P \) discounted at \( r\% \) per period have a present value of \( 100P/r \).)

   b) If your original balance is \$100,000 and your mortgage rate is 7

   c) If you want to prepay some principal immediately at time 0 to shorten this mortgage to 29 years (keeping the same scheduled monthly payment), what should be the amount of your prepayment?

2. **Impact of Relocation**

   According to recent statistics, the US has about 65 million single-family homes, and existing home sales average about 5 million per year.

   a) If mortgage borrowers move randomly at this pace every year, then roughly what percentage of 30-year mortgages actually stay in the home for the full 30 years?

   b) If relocation was the only reason for mortgage prepayments, and long-term investors wanted to earn 7\% yield (compounded annually), roughly how much would they pay today for a mortgage security that gives them the right to receive the 360th monthly payment on your new \$100,000 mortgage from (lb)?

3. **Combining Economic Variables**

   For the average fixed-rate mortgage borrower who currently has principal balance \$B, interest rate \( R\% \), and \( Y_{rem} \) years remaining, we might reasonably define the "real refinancing incentive" (after inflation, taxes, and expenses) as

   \[
   \frac{P_{Index,1984}}{P_{Index, current}} \cdot B \cdot \frac{R - R_{new}}{100} \cdot \left[ (1 - \frac{\text{Tax}}{100}) \cdot \min(Y_{rem}, Y_{be}) - \frac{P\&F}{100} \right]
   \]

   where \( P_{Index} \) is a price index for computing "constant 1984 dollars," \( R_{new} \) is the interest rate for a replacement mortgage, Tax is the marginal income tax rate, P\&F is the percentage "points and fees" for obtaining a new mortgage, and \( Y_{be} \) is the number of years in the average borrower’s "break-even" decision between future interest savings and upfront transaction costs.

   Propose and explain a similar combination of variables to represent the "real relocation incentive." To first-order, assume that the average home-owner:
(i) borrowed a 30-year mortgage on the day they bought the house, with original balance 80% of the original home price;

(ii) sells the house at today’s price level, pays off the current mortgage balance, pays transaction costs totaling 10% of the home sale price, and then uses the remaining cash as the source of a downpayment on the largest-possible new home (in a range Min% to Max% of the original home);

(iii) weighs the benefit of the larger home against its additional after-tax mortgage interest.

4. Link Between Mortgage Rates and Market Interest Rates

In setting the initial rate for new adjustable-rate mortgages, banks take into account the interest rate on market benchmarks (like the 1-year Treasury bill), and also the average interest rate that the bank is paying out on its deposit accounts (primarily saving, checking, and CDs outstanding from the last 1-3 years). Knowing this, and given the historical data (spreadsheet to follow):

a) Construct a sensible model predicting the initial ARM rate from the course of 1-year Treasury rates.

b) If Treasury rates remain unchanged at the November level, what would be your forecast for the initial ARM rate in May 2002, and what would be your level of confidence?