1. The traffic flow can be modeled with \( q = 70 \rho (1 - \rho / 377) \) vehicles per hour. The traffic is moving at a constant speed and a constant density of 250 vehicles/mile. Because of an overturned truck, cars start to slow down slightly, causing a slight density increase. At \( t = 0 \) we thus observe that the density is 250 when \( x < 0 \) and is 270 when \( x > 0 \).

(a) How fast are cars moving before they encounter the wave?
(b) Using linear theory with \( \rho_0 = 250 \), estimate the velocity of the traffic wave. For what \( t \) (in minutes) will the car which was located at \( x = -1 \) mile at \( t = 0 \) encounter the wave? Sketch the wave propagation in an \( x - t \) diagram.
(c) Show from the general theory that a driver in the pack at density 250 will see the traffic wave approaching at speed \( u_{\text{max}} \rho / \rho_{\text{max}} \). Check this against the numbers you gave in (a) and (b).

2. Solve the following linear first-order PDEs with the indicated initial condition. In each case verify that you have a solution by substitution back in the equation.

\[(a) \quad \frac{\partial f}{\partial t} + \frac{xt}{1 + t^2} \frac{\partial f}{\partial x} = 0, \quad f(x, 0) = \sin(x), \]
\[(b) \quad \frac{\partial f}{\partial t} + \frac{1}{1 + x} \frac{\partial f}{\partial x} = 0, \quad f(x, 0) = x. \]

In (b) assume \( x > -1 \). (Hint in (b): \( F(\phi) = -1 + \sqrt{1 + 2\phi} \).)

3. Problem 71.1, page 322 of text. (\( a \) is a positive constant, and \( t \) is measured in hours.)

4. Problem 71.2, page 322 of text.

5. Apply the method we have used to solve the nonlinear traffic flow equation to the equation
\[
\frac{\partial \rho}{\partial t} + \rho^2 \frac{\partial \rho}{\partial x} = 0, \quad x > 0,
\]
with the initial condition \( \rho(x, 0) = x > 0 \). Verify your answer by substituting in the equation. (Hint: \( x = \rho^2(x_0, 0)t + x_0 \).)

6. For the red light problem with \( q = u_{\text{max}} \rho (1 - \rho / \rho_{\text{max}}) \), show that the expansion fan in the transition region has the form
\[
\rho(x, t) = \rho_{\text{max}} \left( \frac{u_{\text{max}} t - x}{2 u_{\text{max}} t} \right).
\]