1. Derive the shock polar relation for oblique shocks,

\[ v_s^2 = \frac{(u_1 u_2 - c_s^2)(u_1 - u_2)^2}{\gamma + 1 u_1^2 - u_1 u_2 + c_s^2}. \]

2. Suppose a steady 2D flow of a polytropic gas at Mach number 2, adjacent to a plane wall, encounters an abrupt bend of the wall by -30°, producing an expansion fan. What is the Mach number downstream of this corner?

3. This is a problem for exploring the interesting phenomenon of sound generated by fluid flow (e.g. the bubbling of a brook or the sound of an air jet). (The pioneering work of Lighthill on this subject is surveyed by Ffowcs-Williams in Ann. Rev. Fluid Dyn. 1, 197-222, which is on reserve.)

To study the sound generated aerodynamically in a fluid, assume that the density is \( \rho = \rho_0 + \rho' \) and similarly for \( p \), where \( \rho_0, p_0 \) are constants and \( \rho' \approx p'/c_0^2 \). For an inviscid gas, show that the equations of mass and momentum may then be combined to obtain an equation of the form

\[ \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = F. \]

Find the form of \( F \) in terms of \( \rho \) and \( u \) and their derivatives. What form is taken by \( F \) if we have \( \rho \approx \rho_0 \) and \( u \) is taken to have zero divergence? (See also L&L §75 for additional results.)