Lecture 10: Dispersion Trading

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What is dispersion trading?

• Dispersion trading refers to trades in which one
  -- sells index options and buys options on the index components, or
  -- buys index options and sells options on the index components

• All trades are delta-neutral (hedged with stock)

• The package is maintained delta-neutral over the horizon of the trade

Dispersion trading:

-- selling index volatility and buying volatility of the index components
-- buying index volatility and selling volatility on the index components
Why Dispersion Trading?

Motivation: to profit from price differences in volatility markets using index options and options on individual stocks

Opportunities: Market segmentation, temporary shifts in correlations between assets, idiosyncratic news on individual stocks
Index Arbitrage versus Dispersion Trading

**Index Arbitrage:**
Reconstruct an index or ETF using the component stocks

**Dispersion Trading:**
Reconstruct an index option using options on the component stocks
Main U.S. indices and sectors

- **Major Indices**: SPX, DJX, NDX
  SPY, DIA, QQQQ (Exchange-Traded Funds)

- **Sector Indices**:
  - Semiconductors: SMH, SOX
  - Biotech: BBH, BTK
  - Pharmaceuticals: PPH, DRG
  - Financials: BKX, XBD, XLF, RKH
  - Oil & Gas: XNG, XOI, OSX
  - High Tech, WWW, Boxes: MSH, HHH, XBD, XCI
  - Retail: RTH
Intuition...

\[ I = \sum_{i=1}^{n} w_i S_i \quad w_i = \text{number of shares in index} \]

\[ \frac{dI}{I} = \frac{1}{I} \sum_{i=1}^{n} w_i dS_i = \sum_{i=1}^{n} \frac{w_i S_i}{I} \frac{dS_i}{S_i} \]

\[ = \sum_{i=1}^{n} p_i \frac{dS_i}{S_i}, \quad p_i = \frac{w_i S_i}{I} \]

\[ \sigma_I^2 = \text{Var}\left\{ \frac{dI}{I} \right\} = \text{Var}\left\{ \sum_{i=1}^{n} p_i \frac{dS_i}{S_i} \right\} \]

\[ = \sum_{ij} p_i p_j \text{Cov}\left\{ \frac{dS_i}{S_i}, \frac{dS_j}{S_j} \right\} \]

\[ \sigma_I^2 = \sum_{ij} p_i p_j \sigma_i \sigma_j \rho_{ij} \]

Fair value relation for volatilities assuming a given correlation matrix
The trade in pictures

Sell index call

Buy calls on different stocks.

Delta-hedge using index and stocks
Profit-loss scenarios for a dispersion trade in a single day

Scenario 1

Stock P/L: - 2.30
Index P/L: - 0.01
Total P/L: - 2.41

Scenario 2

Stock P/L: +9.41
Index P/L: - 0.22
Total P/L: +9.18
First approximation to the dispersion package: "Intrinsic Value Hedge"

\[ I = \sum_{i=1}^{M} w_i S_i \quad w_i = \text{number of shares, scaled by "divisor"} \]

\[ K = \sum_{j=1}^{M} w_i K_i \quad \Rightarrow \]

\[ \max(I - K, 0) \leq \sum_{j=1}^{M} w_i \max(S_i - K_i, 0) \]

\[ C_I(I, K, T) \leq \sum_{j=1}^{M} w_i C_i(S_i, K_i, T) \]

IVH: use index weights for option hedge

IVH: premium from index is less than premium from components “Super-replication”

Makes sense for deep-in-the-money options
Intrinsic-Value Hedging is `exact' only if stocks are perfectly correlated

\[ I(T) = \sum_{i=1}^{M} w_i S_i(T) = \sum_{i=1}^{M} w_i F_i e^{\sigma_i N_i - \frac{1}{2} \sigma_i^2 T} \]

\[ \rho_{ij} \equiv 1 \implies N_i \equiv N = \text{standardized normal} \]

Solve for \( X \) in:

\[ K = \sum_{i=1}^{M} w_i F_i e^{\sigma_i X - \frac{1}{2} \sigma_i^2 T} \]

Set:

\[ K_i = F_i e^{\sigma_i X - \frac{1}{2} \sigma_i^2 T} \]

\[ \therefore \]

\[ \max(I(T) - K, 0) = \sum_{i=1}^{M} w_i \max(S_i(T) - K_i, 0) \quad \forall T \]

Similar to Jamshidian (1989) for pricing bond options in 1-factor model.
IVH : Hedge with "equal-delta" options

\[ K_i = F_i e^{\sigma_i x \sqrt{T} - \frac{1}{2} \sigma^2_i T} \]

\[ \therefore \quad X = \frac{1}{\sigma_i \sqrt{T}} \ln\left( \frac{K_i}{F_i} \right) + \frac{1}{2} \sigma_i \sqrt{T} \]

\[ -X = \frac{1}{\sigma_i \sqrt{T}} \ln\left( \frac{F_i}{K_i} \right) - \frac{1}{2} \sigma_i \sqrt{T} = d_2 \]

\[ N(d_2) = \text{constant} \]

log - moneyness\(\approx\) constant

Deltas \(\approx\) constant
What happens after you enter an option trade?

Profit-loss for a hedged single option position (Black–Scholes)

\[ P / L \approx \theta \cdot (n^2 - 1) + NV \cdot \frac{d\sigma}{\sigma} \]

\[ \theta = \text{time-decay (dollars)}, \quad n = \frac{\Delta S}{S\sigma\sqrt{\Delta t}}, \quad NV = \text{normalized Vega} = \sigma \frac{\partial C}{\partial \sigma} \]

\( n \sim \text{standardized move} \)
Gamma P/L for an Index Option

Assume \( d\sigma = 0 \)

Index Gamma P/L = \( \theta_I \left(n_I^2 - 1\right) \)

\[
n_I = \sum_{i=1}^{M} \frac{p_i \sigma_i n_i}{\sigma_I} \quad \quad p_i = \frac{w_i S_i}{\sum_{j=1}^{M} w_j S_j}
\]

\[
\sigma_i^2 = \sum_{ij=1}^{M} p_i p_j \sigma_i \sigma_j \rho_{ij}
\]

Index P/L = \( \theta_I \sum_{i=1}^{M} \frac{p_i^2 \sigma_i^2}{\sigma_I^2} (n_i^2 - 1) + \theta_I \sum_{i \neq j} \frac{p_i p_j \sigma_i \sigma_j (n_i n_j - \rho_{ij})}{\sigma_I^2} \)
Gamma P/L for Dispersion Trade

\(i^{th} \text{ stock P/L} \approx \theta_i \cdot (n_i^2 - 1)\)

\(\text{Dispersion Trade P/L} \approx \sum_{i=1}^{M} \left( \theta_i + \frac{p_i^2 \sigma_i^2 \theta_i}{\sigma^2} \right) (n_i^2 - 1) + \theta_1 \sum_{i \neq j} \frac{p_i p_j \sigma_i \sigma_j}{\sigma^2} (n_i n_j - \rho_{ij})\)

diagonal term: realized single-stock movements vs. implied volatilities

off-diagonal term: realized cross-market movements vs. implied correlation
Dispersion Statistic

\[ D^2 = \sum_{i=1}^{N} p_i (X_i - Y)^2 \]

\[ X_i = \frac{\Delta S_i}{S_i}, \quad Y = \frac{\Delta I}{I} \]

\[ D^2 = \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 - \sigma_i^2 n_i^2 \]

\[ P/L = \sum_{i=1}^{N} \theta_i (n_i^2 - 1) + \theta_l (n_l^2 - 1) \]

\[ = \sum_{i=1}^{N} \theta_i n_i^2 + \theta_l n_l^2 - \Theta \]

\[ \Theta \equiv \sum_{i=1}^{N} \theta_i + \theta_l \]

\[ = \sum_{i=1}^{N} \theta_i n_i^2 + \frac{\theta_l}{\sigma_l^2} \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 - \frac{\theta_l}{\sigma_l^2} \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 + \theta_l n_l^2 - \Theta \]

\[ = \sum_{i=1}^{N} \left( \frac{\theta_l p_i \sigma_i^2 n_i^2}{\sigma_l^2} + \theta_l \right) n_i^2 - \frac{\theta_l}{\sigma_l^2} D^2 - \Theta \]
Summary of Gamma P/L for Dispersion Trade

Gamma P/L = \[ \sum_{i=1}^{N} \left( \frac{\theta_i p_i \sigma_i^2 n_i^2}{\sigma_i^2} + \theta_i \right) n_i^2 - \frac{\theta_i}{\sigma_i^2} D^2 - \Theta \]

"Idiosyncratic" Gamma
Dispersion Gamma
Time-Decay

Example: "Pure long dispersion" (zero idiosyncratic Gamma):

\[ \theta_i = -\theta_i \frac{p_i \sigma_i^2}{\sigma_i^2} \]
\[ \Theta = \left| \theta_i \right| \left( \sum_{i}^{\prime} \frac{p_i \sigma_i^2}{\sigma_i^2} - 1 \right) \geq \left| \theta_i \right| \left( \sum_{i}^{\prime} \frac{\left( \sum_{i}^{\prime} p_i \sigma_i \right)^2}{\sigma_i^2} - 1 \right) > 0 \]
Payoff function for a trade with short index/long options (IVH), 2 stocks

Value function (B&S) for the IVH position as a function of stock prices (2 stocks)

In general: short index IVH is short-Gamma along the diagonal, long-Gamma for "transversal" moves
Gamma Risk: Negative exposure for ‘parallel’ shifts, positive ‘exposure’ to transverse shifts

\[ \sigma_1 = 30\% \]

\[ \sigma_2 = 40\% \]

\[ \rho_{12} = .5 \]
Gamma-Risk for Baskets

$D = \text{Dispersion, or cross-sectional move, } D/(Y^*Y) = \text{Normalized Dispersion}$

$X_i = \frac{\Delta S_i}{S_i} \quad Y = \frac{\Delta I}{I}$

$D = \sum_{i=1}^{N} p_i (X_i - Y)^2$

$D/Y^2 = \sum_{i=1}^{N} p_i \left( \frac{X_i}{Y} - 1 \right)^2$

From realistic portfolio
Vega Risk

Sensitivity to volatility: perturb all single-stock implied volatilities by the same percent amount

\[
\text{Vega P/L} = \sum_{j=1}^{M} \text{Vega}_j \Delta \sigma_j + \text{Vega}_I \Delta \sigma_I \\
= \sum_{j=1}^{M} (NV)_j \frac{\Delta \sigma_j}{\sigma_j} + (NV)_I \frac{\Delta \sigma_I}{\sigma_I} \\
= \left[ \sum_{j=1}^{M} (NV)_j + (NV)_I \right] \frac{\Delta \sigma}{\sigma}
\]

\[NV = \text{normalized vega} = \sigma \frac{\partial V}{\partial \sigma}\]
Market/Volatility Risk

- Short Gamma on a perfectly correlated move
- Monotone-increasing dependence on volatility (IVH)
``Rega’’: Sensitivity to correlation

\[ \rho_{ij} \rightarrow \rho_{ij} + \Delta \rho \quad i \neq j \]

\[ \sigma_i^2 \rightarrow \sum_{i \neq j} p_i p_j \sigma_i \sigma_j \rho_{ij} + \left( \sum_{i \neq j} p_i p_j \sigma_i \sigma_j \right) \Delta \rho \]

\[ \Delta \sigma_i^2 = \left( \sigma_i^{(1)} \right)^2 - \left( \sigma_i^{(0)} \right)^2 \Delta \rho, \quad \sigma_i^{(1)} = \sum_{j=1}^{M} p_j \sigma_j, \quad \sigma_i^{(0)} = \sqrt{\sum_{j=1}^{M} p_j^2 \sigma_j^2} \]

\[ \frac{\Delta \sigma_i}{\sigma_i} = \frac{1}{2} \left( \frac{\sigma_i^{(1)}}{\sigma_i^{(0)}} \right)^2 \Delta \rho \]

Correlation P/L = \[ \frac{1}{2} (NV) \left( \frac{\sigma_i^{(1)}}{\sigma_i^{(0)}} \right)^2 \Delta \rho \]

Rega = \[ \frac{1}{2} \left( \frac{\sigma_i^{(1)}}{\sigma_i^{(0)}} \right)^2 \times (NV) \]
Market/Correlation Sensitivity

- Short Gamma on a perfectly correlated move
- Monotone-decreasing dependence on correlation
A model for dispersion trading signals (taking into account volatility skews)

• Given an index (DJX, SPX, NDX) construct a proxy for the index with small residual.

\[
\frac{dI}{I} = \sum_{k=1}^{m} \beta_k \frac{dS_k}{S_k} + \epsilon \quad \text{(multiple regression)}
\]

• Alternatively, truncate at a given capitalization level and keep the original weights, modeling the remainder as a stock w/o options.

• Build a Weighted Monte Carlo simulation for the dynamics of the m stocks and value the index options with the model.

• Compare the model values with the bid/offer values for the index options traded in the market.
Morgan Stanley High-Technology 35 Index (MSH)

- 35 Underlying Stocks
- Equal-dollar weighted index, adjusted annually
- Each stock has typically O(30) options over a 1yr horizon
Test problem: 35 tech stocks

Price options on basket of 35 stocks underlying the MSH index

Number of constraints: 876

Number of paths: 10,000 to 30,000 paths

Optimization technique: Quasi-Newton method (explicit gradient)
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Fragment of data for calibration with 876 constraints
Near-month options
(Pricing Date: Dec 2000)

![Graph showing MSH Basket option: model vs. market](image)
Second-month options

Basket option: model vs. market

Graph showing the implied volatility for different strikes, comparing model, midmarket, bid, and offer prices.
Third-month options

Basket option: model vs. market

- **strike**
  - 600
  - 640
  - 680
  - 720
  - 760

- **implied vol**
  - Model
  - Midmarket
  - Bid
  - Offer
Six-month options

Basket option: model vs. market

![Graph showing implied volatility vs. strike for model, midmarket, bid, and offer prices.](image-url)
Broad Market Index Options (OEX)
Pricing Date: Oct 9, 2001

Skew Graph

Bid Price
Ask Price
Model Fair Value
Hedging

- Covering the "wings" in every name implies an excess Vega risk. Intrinsic Value Hedge implies long Volatility.

- Use the WMC sensitivity method (regressions) to determine the best single co-terminal option to use for each component.

- Implement a Theta-Neutral hedge using the most important names with the corresponding Betas.
Simulation for OEX Group: $10MM/ Targeting 1% daily stdev

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<tr>
<th>SIGNALSTRENGTH &gt; threshold</th>
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- Constant-VaR portfolio (1% stdev per day)
- Capital is allocated evenly among signals
- Transaction costs in options/ stock trading included
Simulation for QQQ group
$10MM with 1% target daily stdev

<table>
<thead>
<tr>
<th>signal &gt;threshold</th>
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QQQ; number of signals

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Simulation for QQQ+OEX
$10MM with 1% daily stdev

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<td>1.4</td>
<td>1.7</td>
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</table>
Includes T.C., in options and stock trading
Dispersion Capacity Estimate

- USD 10 MM ~ 100 OEX contracts per day
- If we assume 1000 contracts to be a liquidity limit, capacity is 100 MM just for OEX
- Capacity is probably around 200 MM if we use sectors and Europe
- Dispersion has higher Sharpe Ratio:
  It is an arb strategy based on waiting for profit opportunities