(1) [15 points] Do the following limits exist? If so, evaluate the limit; if not, explain why not.

(a) \( \lim_{x \to 0} \frac{\sin x}{|x|} \)

(b) \( \lim_{h \to 0} \frac{1 - (1/h)}{1 + (1/h)} \)

(c) \( \lim_{x \to 1} f'(x) \) where \( f(x) = \begin{cases} 2x & \text{for } x \leq 1 \\ 1 + x^2 & \text{for } x \geq 1 \end{cases} \)

(2) [25 points] Differentiate the following functions:

(a) \( f(x) = \sin(x^2 + x) \)

(b) \( f(x) = e^x \ln x \)

(c) the function \( y(x) \) defined implicitly by \( y^{1/2} + xy + x^{1/2} = 1 \)

(d) \( f(x) = x^{3x} \)

(e) \( f(x) = \int_x^1 \frac{\cos t}{1 + t^3} \, dt \)

(3) [10 points] Consider the use of Newton’s method to find \( \sqrt{5} \).

(a) In general, Newton’s method is used to find a solution of \( f(x) = 0 \). How should we choose the function \( f \) in this case?

(b) Using your answer to (a) and the initial guess \( x_1 = 2 \), what is the next approximation \( x_2 \)?

(4) [20 points] Consider the function \( f(x) = x/(x^2 + 4) \).

(a) For which \( x \) is it increasing? For which \( x \) is it decreasing?

(b) Identify all local maxima and local minima.

(c) What is its behavior as \( x \to \pm \infty \)?

(d) Sketch the graph.

(5) [10 points] Find the largest possible value of the product \( xy \), given that \( x \) and \( y \) are positive and \( x + 2y = 20 \).

(6) [15 points] Find the following indefinite integrals:

(a) \( \int \sin(x) \cos(x) \, dx \)

(b) \( \int x(2x + 1)^{-9} \, dx \)

(c) \( \int x^2 e^x \, dx \)
(7) [10 points] Find the following definite integrals:

(a) \( \int_0^1 \frac{x^2}{x^3 + 1} \, dx \)

(b) \( \int_1^2 x^2 \ln x \, dx \)

(8) [20 points] Consider the region \( R \) in the \( x, y \) plane bounded by the graphs of \( y = 2 \sqrt{x} \) and \( y = 2x^2 \) (see the figure). For each of the following, your answer should be a specific definite integral which you could easily evaluate. However you are not being asked to evaluate it.

(a) Express the area of \( R \) as an integral with respect to \( x \) (in other words, find the area using slices parallel to the \( y \) axis).

(b) Express the area of \( R \) as an integral with respect to \( y \) (in other words, find area using slices parallel to the \( x \) axis).

(c) Now consider the solid of revolution obtained by rotating this region about the \( y \) axis. Express its volume using the method of washers (also called the method of disks or slices).

(d) Still considering the same solid (obtained by rotating \( R \) about the \( y \) axis), express its volume using the method of shells.

(9) [15 points] You are growing bacteria in culture. Assume they reproduce at a constant rate, so the population grows exponentially. Suppose the initial population is 1000, and after 30 minutes you observe that the population is 2000.

(a) When will the population be 4000?

(b) Give a formula for the population at time \( t \), for any time \( t \).

(c) When will the population be 40,000?

(10) [10 points] Answer either Question A or Question B. (You will not get extra credit for answering both.)

**Question A** Consider a positive, increasing function \( f(x) \). In defining the integral \( \int_0^1 f(x) \, dx \) we considered the lower sum \( L \) and the upper sum \( U \) associated with a partition of \([0, 1]\). Suppose the partition is evenly spaced in intervals of \( 1/5 \), i.e., it consists of \( \{0, 1/5, 2/5, \ldots, 4/5, 1\} \). Show that the difference between the upper sum and the lower sum is exactly \( U - L = \frac{4}{5}[f(1) - f(0)] \).

**Question B** Use the mean value theorem to explain why if \( f \) is differentiable and \( f' > 0 \) then \( f \) is an increasing function.