Problemset 2. Due April 11.

The Gamma process is defined as a process with independent and stationary increments whose distribution at time 1 is the exponential distribution with density

\[ p(x) = \begin{cases} 
  e^{-x}dx & \text{if } x \geq 0 \\
  0 & \text{otherwise}
\end{cases} \]

1) What is its distribution at time \( t > 0 \)?

2) What is the Levy-Khintchine representation for the process?

3) Show that the process is increasing and made up only of positive jumps.

4) What is the distribution of the biggest jump during \( 0 \leq t \leq 1 \)?

5) Given \( X(1) \geq A \) what is \( EX(1) \)?

6) If \( A \) is large, show that the large value of \( X(1) \) is due to at least one jump of order of magnitude \( A \) i.e if \( \Omega_{\delta A} \) is the set of paths with no jumps of size larger than \( \delta A \), then

\[ \lim_{\delta \to 0} \limsup_{A \to \infty} P[\Omega_{\delta A}|X(1) = A] = 0 \]

7) If \( Y \) is the largest jump can you calculate asymptotically the conditional distribution of \( \frac{Y}{X(1)} \) given \( X(1) = A \), asymptotically as \( A \to \infty \)?

8) At what point in time would the largest jump have occurred?