Homework 8

1. Find a basis of the integers $\mathcal{O}_K$ in $K = \mathbb{Q}(\sqrt{17}, \sqrt{-19})$.
2. Let $K = \mathbb{Q}(\sqrt[3]{a}, \sqrt[4]{b})$, with $a \neq m^3$, i.e., a is an integer which is not a cube, and $b \neq \pm n^2$. Determine the Galois group $\mathrm{Gal}(\tilde{K}/\mathbb{Q})$ of the Galois closure $\tilde{K}$ of K.
3. Show that $e_1 := 1 + \zeta + \zeta^6$ and $e_2 = 1 + \zeta^3 + \zeta^4$, where $\zeta = \zeta_7 = e^{2\pi i/7}$ is the 7-th root of 1, are multiplicatively independent units in the cyclotomic extension $\mathbb{Q}(\zeta)/\mathbb{Q}$.
4. Determine generators of $\mathcal{O}_K$, where $K = \mathbb{Q}(\sqrt[3]{d})$, with $d \in \mathbb{Z}$.
5. Show that the discriminant of $\mathcal{O}_K$, with $K = \mathbb{Q}(\zeta_p)$, is given by

$$(-1)^{\frac{p-1}{2}} p^{p-2}.$$