Homework 4

1. Show that
$$\sum_{d|n} |\mu(d)| = 2^k,$$
where k is the number of distinct primes dividing n.

2. Show that
$$\sum_{n\geq 1} -\frac{\mu(n)}{n} \cdot \log(1-x^n) = x$$
$$\sum_{n\geq 1,\, p\nmid n} -\frac{\mu(n)}{n} \cdot \log(1-x^n) = x + \frac{x^p}{p} + \frac{x^{p^2}}{p^2} + \cdots$$

3. Show that $f(x) = x^p - x - 1 \in \mathbb{Z}_p[x]$ is irreducible.

4. Compute the first p coefficients of $E_p(x)$.

5. Consider
$$f(x) := 1 - (x/2)^2/1!^2 + (x/2)^4/2!^2 - (x/2)^6/3!^2 + \cdots$$
When does this converge for $x \in \mathbb{R}$? When does this converge for x in the p-adics, with odd p?