

### Homework 3

1. Compute  $\sqrt{-1}$  in  $\mathbb{Q}_{29}$  to 5 digits.
2. For  $c \in \mathbb{N}$  show that the sequence  $c_n := c^{p^n}$  converges in  $\mathbb{Q}_p$ . Let  $\gamma = \lim c_n$ . Then  $\gamma = c \pmod{p}$  and  $\gamma^{p-1} = 1$ .
3. Show that  $\mathbb{Q}_p$  contains all  $(p-1)$ st roots of 1.
4. Let  $f(x) := \sum_n a_n x^n \in \mathbb{Q}_p[[x]]$  and let  $r = r(f)$  be its convergence radius. Show that  $r(f) \leq r(f')$ . Give an example where  $r(f) < r(f')$ .
5. Show that  $\mathbb{Q}_p$  admits no automorphisms.