**Homework 2**

1. What are the last 3 digits (in decimals) of
$$3^{999} - 2^{999}?$$

2. Show that if $x^2 + y^2 = z^2$, for some positive integers $x, y, z$ then $xyz$ is divisible by 60.

3. Find all pairs of positive integers $x, y$ such that $x^4 + 4y^4$ is a prime number.

4. Decide (and explain) whether the equation
$$x_1^3 + x_2^5 + x_3^7 + x_4^{11} = x_5^{13}$$
has no, or finitely many, or infinitely many solutions in positive integers.

5. Show that
$$3x^3 + 4y^3 + 5z^3 = 0$$
has a nontrivial solution modulo $p$, for all primes $p$.