Homework 1

1. Show that for all $x \in \mathbf{R}, m \in \mathbf{N}$ one has

$$[\frac{[x]}{m}] = [\frac{x}{m}].$$

2. Show that, for $a, b, c \in \mathbf{R}$, if all three of the following are positive, then so are a, b, c:

$$a + b + c, \quad ab + bc + ca, \quad abc.$$

3. Find all integers n such that

$$(n + 4) \mid n^2 + 8n + 15.$$

4. Find all positive integral solutions of

$$x! + y! + z! = u!.$$

5. Find all integral solutions of the equation

$$x(x + 1)(x + 7)(x + 8) = y^2.$$