

### Homework 4 / due October 10

1. Find a normal subgroup of  $\mathfrak{S}_4$  of order 4.
2. Show that a group of order 495 is solvable.
3. Write

$$(x^2 + y^2)(x^2 + z^2)(y^2 + z^2)$$

in terms of elementary symmetric functions  $\sigma_1, \sigma_2, \sigma_3$ .

4. Determine the ring of invariants  $\mathbb{C}[x, y, z]^\Gamma$  for

$$\Gamma := \left\{ \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{pmatrix} \right\} \subset \mathrm{GL}_3(\mathbb{C}).$$

5. Determine the ring of invariants  $\mathbb{F}_2[x, y, z]^\Gamma$  for

$$\Gamma := \left\{ \begin{pmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{pmatrix} \right\} \subset \mathrm{GL}_3(\mathbb{F}_2),$$

where  $*$  is 0 or 1, i.e.,  $\Gamma$  is the Heisenberg group over  $\mathbb{F}_2$ .