

Homework 4 / due October 10

1. Find a normal subgroup of \mathfrak{S}_4 of order 4.

2. Show that a group of order 495 is solvable.

3. Write

$$(x^2 + y^2)(x^2 + z^2)(y^2 + z^2)$$

in terms of elementary symmetric functions $\sigma_1, \sigma_2, \sigma_3$.

4. Determine the ring of invariants $\mathbb{C}[x, y, z]^\Gamma$ for

$$\Gamma := \left\{ \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{pmatrix} \right\} \subset \mathrm{GL}_3(\mathbb{C}).$$

5. Determine the ring of invariants $\mathbb{F}_2[x, y, z]^\Gamma$ for

$$\Gamma := \left\{ \begin{pmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{pmatrix} \right\} \subset \mathrm{GL}_3(\mathbb{F}_2),$$

where $*$ is 0 or 1, i.e., Γ is the Heisenberg group over \mathbb{F}_2 .