Homework 1 (due September 19)

1. Determine $d := \gcd(4655, 12075)$ and find $x, y \in \mathbb{Z}$ such that
$$d = 4655x + 12075y.$$

2. Let $p > 2$ be a prime number. Show that
$$1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{p-2} = \frac{2^{p-1} - 1}{p} \quad \mod p.$$

3. Prove that an integer of the form $8n + 7$ cannot be written as a sum of three integer squares.

4. Let $(m, n) = 1$. Then
$$\frac{(m+n-1)!}{m!n!} \in \mathbb{Z}.$$

5. For every $n \in \mathbb{N}$ there exists an $m \in \mathbb{Z}$ such that
$$(\sqrt{2} - 1)^n = \sqrt{m+1} - \sqrt{m}.$$