Homework 6

1. Is it possible to construct (with ruler and compass) a square whose area is equal to the area of a given triangle?

2. Let $K = \mathbb{Q}(\alpha)$ where $\alpha = \sqrt[3]{a}$, with $a \in \mathbb{Z}$, a squarefree. Show that if $a \neq \pm 1 \mod 9$ then $\mathcal{O}_K = \mathbb{Z} + \mathbb{Z}\alpha + \mathbb{Z}\alpha^2$.

3. Find an integral basis for $\mathcal{O}_K$, where $K = \mathbb{Q}(\alpha)$ and $\alpha^3 - \alpha + 1 = 0$.

4. Let $K = \mathbb{Q}(\alpha)$, where α is a root of $x^3 - x - 1$. Find the irreducible polynomial for $\gamma := 1 + \alpha^2$ over $\mathbb{Q}$.

5. Let $\mathfrak{a}$ be an integral ideal in $\mathcal{O}_K$. Then

$$\cap_{n=1}^{\infty} \mathfrak{a}^n = \begin{cases} \mathcal{O}_K & \text{if } \mathfrak{a} = \mathcal{O}_K \\ (0) & \text{otherwise} \end{cases}$$