

Homework 5

1. Does there exist a normal extension $L \supset \mathbb{Q}(\sqrt{3}) \supset \mathbb{Q}$ with cyclic Galois group $\text{Gal}(L/\mathbb{Q}) = \mathbb{Z}/4\mathbb{Z}$?
2. Determine all subfields of $\mathbb{Q}(\zeta_9)$.
3. Determine a polynomial $f \in \mathbb{Z}[x]$ whose roots generate a cyclic extension of \mathbb{Q} of order 5.
4. Show that $K := \mathbb{Q}(\sqrt{2 + \sqrt{-5}})$ is normal over \mathbb{Q} and determine its Galois group.
5. Show that it may happen that $[LK : K] < [L : L \cap K]$.