

### Homework 5

1. Does there exist a normal extension  $L \supset \mathbb{Q}(\sqrt{3}) \supset \mathbb{Q}$  with cyclic Galois group  $\text{Gal}(L/\mathbb{Q}) = \mathbb{Z}/4\mathbb{Z}$ ?
2. Determine all subfields of  $\mathbb{Q}(\zeta_9)$ .
3. Determine a polynomial  $f \in \mathbb{Z}[x]$  whose roots generate a cyclic extension of  $\mathbb{Q}$  of order 5.
4. Show that  $K := \mathbb{Q}(\sqrt{2 + \sqrt{-5}})$  is normal over  $\mathbb{Q}$  and determine its Galois group.
5. Show that it may happen that  $[LK : K] < [L : L \cap K]$ .