Homework 4 / due October 9

1. Find a normal subgroup of $\mathfrak{S}_4$ of order 4.
2. Show that a group of order 385 is solvable.
3. Write
$$(x^2 + y^2)(x^2 + z^2)(y^2 + z^2)$$
in terms of elementary symmetric functions $\sigma_1, \sigma_2, \sigma_3$.
4. Determine the ring of invariants $\mathbb{C}[x, y, z]^\Gamma$ for
$$\Gamma := \{ \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{pmatrix} \} \subset \mathrm{GL}_3(\mathbb{C}).$$
5. Find generators of the ring of invariants $\mathbb{F}_2[x, y, z]^\Gamma$ for
$$\Gamma := \{ \begin{pmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{pmatrix} \} \subset \mathrm{GL}_3(\mathbb{F}_2),$$
where $*$ is 0 or 1, i.e., Γ is the Heisenberg group over $\mathbb{F}_2$.