Homework 3

1. Determine all groups of order 18.

2. Let p be a prime number. What is the order of $SL_2(\mathbb{Z}/p\mathbb{Z})$?

3. What is the index $(SL_2(\mathbb{Z}/p\mathbb{Z}) : \Gamma_0(p))$?

4. Realize $\mathbb{Z}/3\mathbb{Z}$, $\mathbb{Z}/4\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$ as subgroups of $GL_2(\mathbb{Z})$.

5. Find all subgroups of the symmetric group $\mathfrak{S}_4$ of order 8.

6. Assume that G is generated by two elements and that $\exp(G) = 3$, i.e., for every $g \in G$, $g^3 = 1$. Show that G is finite.