Homework 2

1. Let $p = 1 \pmod 4$ be a prime number. Then
$$\sum_{a=1}^{p-1} \left(\frac{a}{p}\right) a = 0.$$

2. Let $p > 5$ be a prime. Show that
$$\sum_{a=1}^{p-1} \left(\frac{a}{p}\right) a^2 = 0 \pmod p.$$

3. For p prime, $p \nmid b$
$$\sum_{a=1}^{p-1} \left(\frac{a(a+b)}{p}\right) = -1.$$

4. Find the number of non-trivial solutions of
$$x^3 + y^3 + z^3 + t^3 = 0 \pmod 5.$$

5. Show that the congruence
$$x^4 - 17y^4 \equiv 2z^2 \pmod p$$
has nontrivial solutions, for all primes p.