\[ f(x) = x^2 e^x \]
\[ f'(x) = 2xe^x + xe^x = e^x (x^2 + 2x) = xe^x (x + 2). \]
\[ f''(x) = e^x (x^2 + 2x) + e^x (2x + 2) = e^x (x^2 + 4x + 2). \]

\[
\begin{array}{c|cccccc}
\text{sign}(f') & + & + & + & + & + & + \\
\text{sign}(f'') & + & + & + & - & - & + \\
\end{array}
\]

Thus \( f' \) is increasing on \((-\infty, -2) \cup (0, \infty)\) and decreasing on \((-2, 0)\).

Thus \( f \) has a local max at \( x = -2 \) and a local min at \( x = 0 \).

Also \( f'' \) is concave up on \(( -\infty, -2 - \sqrt{2}) \cup (-2 + \sqrt{2}, \infty)\) and concave down on \((-2 - \sqrt{2}, -2 + \sqrt{2})\).

So \( f \) has no inflection points at \(-2 \pm \sqrt{2}\).
The denominator is bounded away from zero, so there are no vertical 
assymptotes.
\[
\lim_{x \to +\infty} \frac{e^x}{1 + e^x} = \lim_{x \to +\infty} \frac{e^x}{e^x + e^x} = \frac{1}{0 + 1} = 1,
\]
so \( f \) has a Horizontal 
assymptote of \( y = 1 \) in the positive direction.
\[
\lim_{x \to -\infty} \frac{e^x}{1 + e^x} = \lim_{x \to -\infty} \frac{e^x}{e^x + e^x} = 0,
\]
so \( f \) has a horizontal 
assymptote of \( y = 0 \) in the negative direction.
\[
f'(x) = \frac{e^x (1 + e^x) - e^x}{(1 + e^x)^2} = \frac{e^x}{(1 + e^x)^2} > 0 \text{ for all } x, \text{ so } f \text{ is always increasing.}
\]
\[
f''(x) = \frac{e^x (1 + e^x)^2 - 2(1 + e^x) e^x}{(1 + e^x)^4} = \frac{-e^x (e^x - 1)}{(e^x + 1)^3} \text{ sign } (f'') \frac{\uparrow \downarrow}{\uparrow \downarrow} 0
\]
So \( f \) has no local extrema 
and one inflection point at 
\( x = 0 \).
Consider \( f(x) = x^3 \) and \( c = 0 \).

Then \( f''(x) = 3x^2 \), \( f'''(x) = 6x \), \( f''''(x) = 6 \).

Thus \( f'(c) = 0 = f''(c) ; f'''(c) > 0 \), but \( f \) has neither a local max nor a local min at \( c \).

Now for any arbitrary \( f \) which satisfies the hypotheses, define \( f''''(x) = g(x) \). Then \( g \) is continuous and \( g'(c) > 0 \) and \( g(c) = 0 \). Thus for sufficiently small \( \varepsilon > 0 \), \( g(c+\varepsilon) > 0 \) and \( g(c-\varepsilon) < 0 \). Thus \( f''''(c+\varepsilon) > 0 \) and \( f''''(c-\varepsilon) < 0 \) so that \( f'''' \) changes sign at \( c \), so that \( f \) has an inflection point at \( c \).
\[ f(x) = \frac{x^2}{x^2 + 9} \]

Note \( f(-x) = \frac{(-x)^2}{(-x)^2 + 9} = \frac{x^2}{x^2 + 9} = f(x) \), so \( f \) is even, not odd, and not periodic since \( f \) is a non-constant rational function.

The \( x \) intercept is the solution of \( \frac{x^2}{x^2 + 9} = 0 \Rightarrow x = 0 \), and \( f(0) = 0 \), so the \( x \) and \( y \) intercepts occur at \((0,0)\).

Note that the denominator is bounded away from zero so \( f \) has no vertical asymptotes.

\[
\lim_{x \to \pm \infty} f(x) = \lim_{x \to \pm \infty} \frac{x^2}{x^2 + 9} = \lim_{x \to \pm \infty} \frac{\frac{1}{x^2}}{1 + \frac{9}{x^2}} = \frac{1}{1 + 0} = 1.
\]

So in the positive direction, \( f \) has a horizontal asymptote of 1. Since \( f \) is even, it also has a horizontal asymptote of 1 in the negative direction.

\[
f''(x) = \frac{2x(x^2 + 9) - 2x(x^2)}{(x^2 + 9)^2} = \frac{2x^3 + 18x - 2x^3}{(x^2 + 9)^2}
\]

\[
\text{Sign}(f) = \begin{array}{cccccc}
\uparrow & \uparrow & \uparrow & \uparrow & \downarrow & \downarrow & \cdots & \downarrow & \downarrow & \uparrow & \uparrow & \uparrow & \uparrow
\end{array}
\]
so \( f \) is increasing on \((0, \infty)\) and decreasing on \((-\infty, 0)\). Thus \( f \) has a local min at \( x = 0 \).

\[
f''(x) = \left( \frac{8}{(x^2+9)^2} \right)' = \frac{18(x^2+9)^2 - 7(x^2+9)(2x)(18x)}{(x^2+9)^4} = \frac{18(x^2+9) - 4 \cdot 18 \cdot x^2}{(x^2+9)^4} = \frac{-3 \cdot 18 \cdot x^2 + 18}{(x^2+9)^4} = 0
\]

\[\Leftrightarrow \quad x = \pm \sqrt{3}.
\]

\[
\text{sign}\ (f'') = \begin{array}{c|c|c|c}
- & + & + & - \\
-\sqrt{3} & \sqrt{3} & \\
\end{array}
\]

So \( f \) is concave up on \((-\sqrt{3}, \sqrt{3})\) and concave down on \((-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)\).

Thus \( f \) has inflection points at \( x = \pm \sqrt{3} \).
\[ f(x) = \frac{x^3 - 1}{x^2 + 1}. \]

The x-intercepts are the solutions of \( \frac{x^3 - 1}{x^2 + 1} = 0 \iff x = 1. \)

So \( x_{int} = 1. \) \( y_{int} = f(0) = -1. \)

\[ f(-x) = \frac{(-x)^3 - 1}{(-x)^2 + 1} = \frac{-x^3 - 1}{-x^2 + 1} \neq f(x) \]

and \( f(-x) \neq -f(x). \)

So \( f \) is neither even, nor odd, nor periodic.

\( f \) is defined and continuous for all \( x \in \mathbb{R} \setminus \{1\}. \)

\[ \lim_{x \to -1^{-}} f(x) = \lim_{x \to -1^{-}} \frac{x^3 - 1}{x^2 + 1} = -\infty \]

\[ \lim_{x \to -1^{+}} f(x) = \lim_{x \to -1^{+}} \frac{x^3 - 1}{x^2 + 1} = +\infty. \]

So \( f \) has a vertical asymptote at \( x = -1. \)

\[ \lim_{x \to \pm \infty} f(x) = \lim_{x \to \pm \infty} \frac{x^3 - 1}{x^2 + 1} = \lim_{x \to \pm \infty} \frac{\frac{x^3}{x^3}}{\frac{x^2}{x^3} + \frac{1}{x^3}} = \lim_{x \to \pm \infty} \frac{1 - \frac{1}{x^3}}{1 + \frac{1}{x^3}} = \frac{1}{0}. \]

So \( f \) has a horizontal asymptote of \( y = 1 \) in the positive direction. Similarly, \( \lim_{x \to -\infty} \frac{x^3 - 1}{x^2 + 1} = \lim_{x \to -\infty} \frac{x^3}{x^3 + \frac{1}{x^3}} = 1. \)

So \( f \) has a horizontal asymptote of \( y = 1 \) in the negative direction.
\[ f(x) = \left( \frac{x^2-1}{x+1} \right)' = \frac{3x^2(x^3+1) - 3x^2(x^3-1)}{(x^3+1)^2} = \frac{6x^2}{(x^3+1)^2} = 0 \implies x = 0. \]

**Sign of \( f'(x) \):**

\[ \frac{6x^2}{(x^3+1)^2} \]

so \( f \) is always increasing.

\[ f''(x) = \frac{12x(x^3+1)^2 - 2(x^3+1)(3x^2)(6x^2)}{(x^3+1)^4} \]

\[ = \frac{12x^4 + 12x - 36x^4}{(x^3+1)^3} = \frac{-12x(2x^3-1)}{(x^3+1)^3} = 0 \]

\( \implies \) \( x(2x^3-1) = 0 \)

\( \implies \) \( x = 0 \) or \( x = \left( \frac{1}{2} \right)^{-1/3} \)

**Sign of \( f''(x) \):**

\[ \frac{6x^2}{(x^3+1)^2} \]

so \( f \) is concave up on \( (-\infty, -1) \cup (0, (\frac{1}{2})^{-1/3}) \) and concave down on \( (-1, 0) \cup ((\frac{1}{2})^{-1/3}, \infty) \).

Thus \( f \) has an inflection point at \( x = 0 \) and \( x = (\frac{1}{2})^{-1/3} \).