Geometry Seminar
November 19, 2002, Tuesday, 6:00 p.m.
Room 613, Courant Institute
251 Mercer Street, New York

Long monotone paths in line arrangements

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Abstract

Let $L = \{\ell_1, \ldots, \ell_n\}$ be a set of n given lines in R^2 . A path in the arrangement $A(L)$ is a simple polygonal chain joining a set of distinct vertices in $V = \{\ell_i \cap \ell_j, i < j\}$ by segments which are on lines in L. The length of a path is one plus the number of vertices in V at which the path turns. A path is monotone along a line ax+by=c if its sequence of vertices is monotone when projected orthogonally onto the line. An interesting open question asks for the value of λ_n, the maximal monotone path length that can occur in an arrangement of n lines. Clearly $\lambda_n \leq \binom{n}{2}$.

I will explain the construction of an arrangement of n lines in which there is a monotone path of length $\Omega(n^2/C^{\sqrt{(\log n)}})$, $C > 1$ a suitable constant. This improves an earlier bound of $\Omega(n^{7/4})$.

Joint work with J. Balogh, O.Regev, C. Smyth, M. Szegedy.

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